

Math 215
Homework Set 5: §§15.7–15.8
Fall 2009

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 6th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

- 15.7.3: Consider a function of one variable, $f(x)$, which is continuous on an interval I . If f has a single critical point, $x = a$, in I , then if that point is a local maximum (or minimum) it must also be an absolute maximum. Explain why this is. Next, consider the function of two variables, $g(x, y) = 3xe^y - x^3 - e^{3y}$. Note that this is continuous for all values of x and y .
- (a) Show that $g(x, y)$ has a single critical point, and find it.
 - (b) Show that $g(x, y)$ has a local maximum at this point.
 - (c) Show that this is not the absolute maximum attained by $g(x, y)$.
 - (d) Use *Maple* to produce a graph that shows why this is the case, and explain how this is different from the case of a function of one variable.

- 15.8.1: The Cobb-Douglas model for the production output from a company is

$$P(L, K) = b L^\alpha K^{1-\alpha},$$

where P is the total production (the monetary value of all goods produced in a year), L is the amount of labor (the total number of person-hours worked in a year), and K is the amount of capital invested (the monetary worth of all machinery, equipment and buildings). The constants b and α are positive, with $\alpha < 1$. The model follows from certain assumptions about the economy (as are described in §15.3 in the text).

- (a) If the cost of a unit of labor is m and the cost of a unit of capital is n , then, if the company's budget is p dollars, there is the constraint $p = mL + nK$. Show that the maximum production occurs when $L = \alpha p/m$ and $K = (1 - \alpha)p/n$.
 - (b) Suppose next that we fix production at Q , so that $bL^\alpha K^{1-\alpha} = Q$, where Q is a constant. In this case what values of L and K minimize the cost function $C(L, K) = mL + nK$?
- 15.8.2: Problem #26 from §15.8.
- 15.8.3: The plane $4x - 3y + 8z = 5$ intersects the cone $z^2 = x^2 + y^2$ in an ellipse.
- (a) Use *Maple* to graph the plane and cone. On the graph, show the ellipse and indicate the highest and lowest points on the ellipse.
 - (b) Use LaGrange Multipliers to find the highest and lowest points.
- 15.8.4: A classic single-variable calculus optimization problem is the following: "The US Postal Service will mail a rectangular box if the sum of its length and girth (the perimeter of a cross-section perpendicular to its length) is at most 108 in. Find the dimensions of such a package with the largest volume." Solve this using Lagrange Multipliers.

M.4: *Maple* problem 4. Consider $f(x, y) = x^3 y$, constrained to $x^2 + 2y^2 = 8$.

- (a) Use *Maple* to plot $z = f(x, y)$. Print out your plot and add the constraint (or use the sneaky trick shown in Example 4 in the `03levelcurves.mw` lab worksheet to plot the constraint with the surface). By hand, indicate where on your graph $f(x, y)$ reaches extreme values on the constraint.
- (b) Also using *Maple*, plot the gradient field plot and contour map for $f(x, y)$, together with the constraint. Show the extremal points on this graph as well. Explain how this graph tells you where the (local) maximum and minimum values of $f(x, y)$ will be on the constraint.
- (c) Use Lagrange Multipliers to find the maximal and minimal values of $f(x, y)$ on the constraint. Confirm that the values you get are consistent with your observations in (a) and (b).