Math 215
Homework Set 6: §§16.1–16.6
Fall 2009

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 6th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

16.1.1: The integral $\iint_D \sqrt{16 - y^2}\, dA$, where $D = [0, 3] \times [-1, 2]$, represents the volume of a solid. Sketch the solid, and explain how you know it is correct.

16.2.1: Sketch a graph of $f(x, y) = \frac{y}{x^2+1}$ for $-2 \le x \le 2$ and $0 \le y \le 1$. Represent the volume under $f(x, y)$ and above the xy-plane as an iterated integral, and find the volume.

16.3.1: Problem #44 from §16.3.

16.3.2: Problem #52 from §16.3.

16.4.1: Sketch the region whose area is given by the integral

$$\int_0^{3\pi/4} \int_0^{4\sin\theta} r\, dr\, d\theta,$$

and then evaluate the integral to find the area.

16.4.2: Problem #30 from §16.4.

16.4.3: Problem #36 from §16.4.

16.5.1: Consider the lamina bounded above by $y = \sqrt{4 - x^2}$, and below by $y = 0$ (for $-2 \le x \le -1$ and $1 \le x \le 2$) and $y = \sqrt{1 - x^2}$. Suppose that the density of the material in the lamina at any point inversely proportional to the point's distance from the origin. Find the center of mass of the lamina.

16.5.2: Alex and Chris meet after their 11 A.M.–noon class every day. Alex' arrival time is X, and Chris' is Y, where X and Y are measured in minutes after noon. The individual density functions for X and Y are

$$f_{\text{alex}}(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & x < 0, \end{cases} \qquad \text{and} \qquad f_{\text{chris}}(y) = \begin{cases} \frac{1}{50}\, y, & 0 \le y \le 10 \\ 0, & \text{otherwise.} \end{cases}$$

(Alex arrives sometime after noon, more likely promptly than late; Chris arrives no later than 12:10 P.M., but more likely late than on time.) If Alex arrives first and Chris is not there, Alex will leave immediately. If Chris arrives first and Alex is not there, Chris will wait until Alex arrives or 12:30, whichever comes first, before leaving. What is the probability that they meet?

16.6.1: Sketch the region over which the integral $\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z)\, dz\, dy\, dx$ is taken. Then write the integral in the other five orders.