

**Math 215**  
**Homework Set 7: §§16.7–17.1**  
**Fall 2009**

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 6th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

16.7.1: Sketch the solid described by the inequalities (in cylindrical coordinates)  $0 \leq \theta \leq \pi/2$ ,  $r \leq z \leq 2$ .

16.7.2: Sketch the solid whose volume is given by the integral  $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} r \, dz \, dr \, d\theta$ . Then evaluate the integral to find the volume.

16.8.1: Sketch the solid described by the inequalities (in spherical coordinates)  $\rho \leq 2$ ,  $\rho \leq \csc \phi$ .

16.8.2: Problem #36 in §16.8.

16.8.3: Show that

$$\int_0^\infty \int_0^\infty \int_0^\infty \sqrt{x^2 + y^2 + z^2} e^{-(x^2+y^2+z^2)} \, dx \, dy \, dz = 2\pi.$$

(The improper integral is defined as the limit of a triple integral over a solid sphere as the radius of the sphere increases indefinitely; c.f., problem #16.4.3 (#36 in §16.4), where we considered the case of the double integral.)

17.1.1: Problems #15–18 in §17.1. In each case, explain your answers.

17.1.2: Consider the vector field  $\mathbf{F}(x, y) = y \mathbf{i} + \mathbf{j}$ .

- (a) Sketch  $\mathbf{F}$  for  $0 \leq x \leq 3$ ,  $0 \leq y \leq 3$ , including vectors at integer points in that domain.
- (b) Sketch flow lines starting at  $(0, 0)$ ,  $(1, 0)$ , and  $(2, 0)$ . What shape do they have? What type of function do you think describes them?
- (c) Show given the definition of  $\mathbf{F}$  that the slope on any flow line is given by  $\frac{dy}{dx} = \frac{1}{y}$ .
- (d) Solve this differential equation to find the equation of the flow line that goes through the origin. Is this consistent with your guess in (b)?

M.5: *Maple* problem 5. At time  $t = 1$  a particle is located at position  $\langle 1, 3 \rangle$ . Suppose that it moves in the velocity field

$$\mathbf{F}(x, y) = \langle xy - 2, y^2 - 10 \rangle.$$

- (a) Find the approximate location of the particle at time  $t = 1.05$ . (You shouldn't use *Maple* for this part of the problem.)
- (b) Use *Maple* to plot this velocity field. As time gets larger, where does the particle go?
- (c) What is the final location that the particle will end up at? Explain how you know this (based on your *Maple* graph, you should be able to find the exact coordinates), and how it relates to the velocity field plot you found in (b).