Math 215
Homework Set 8: §§17.2–17.5
Fall 2009

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 6th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

17.2.1: A thin wire is bent into a circular arc with radius 2, extending from $(2,0)$ to $(\sqrt{2},\sqrt{2})$. If the density of the wire is $\rho(x,y)=kxy$, find the mass and center of mass of the wire.

17.2.2: Problem #47 in §17.2. Explain how you obtain your answer.

17.3.1: Problems #23 and 24 in §17.3.

17.3.2: Let $\mathbf{F}=\nabla f$, where $f(x,y)=\sin(2x+y)$. Find curves C_1 and C_2 that are not closed such that $\int_{C_1}\mathbf{F}\cdot d\mathbf{r}=0$ and $\int_{C_2}\mathbf{F}\cdot d\mathbf{r}=1$. Explain why you pick the curves you do, and how you know that the integrals have the correct values.

17.4.1: Let $P(x,y)=x-2x^2y$, $Q(x,y)=xy^2$, and C be the circle $x^2+y^2=4$. Find $\oint_C P\,dx+Q\,dy$ first by evaluating the line integral and second by using Green's Theorem.

17.4.2: Recall that the centroid of a two dimensional region D is the center of mass when the density is constant, so that the mass is proportional to area, and the center of mass is then given by $\overline{x}=\frac{1}{A}\int_D x\,dA$ and $\overline{y}=\frac{1}{A}\int_D y\,dA$.

(a) Show how you can pick P and Q in Green's theorem so that the expressions become $\overline{x}=\frac{1}{2A}\oint_C x^2\,dy$ and $\overline{y}=-\frac{1}{2A}\oint_C y^2\,dx$, where C is the curve giving the boundary of the region D.

(b) Use this result to find the centroid of the triangular region with vertices $(0,0)$, $(a,0)$, and $(0,a)$.

17.5.1: Problem #12 from §17.5.

17.5.2: Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\operatorname{curl}\mathbf{G}=\langle x^2yz,-y^2z,yz^2\rangle$? $\operatorname{curl}\mathbf{G}=\langle 0,-y^2z,yz^2\rangle$? Explain. In either case, if there is such a vector field, find $\mathbf{G}$. [Hint: can you take a component or components of $\mathbf{G}$ to be zero?]

17.5.3: Let $\mathbf{r}=x\,\mathbf{i}+y\,\mathbf{j}+z\,\mathbf{k}$ and $r=|\mathbf{r}|$. Show that (a) $\nabla\cdot\mathbf{r}=3$; (b) $\nabla\times\mathbf{r}=\overrightarrow{0}$; (c) $\nabla\cdot(r\,\mathbf{r})=4r$; and (d) $\nabla(1/r)=-\mathbf{r}/r^3$.

M.6: *Maple* problem 6. Consider the vector field $\mathbf{F}(x,y)=(x-y)\,\mathbf{i}+xy\,\mathbf{j}$ and the curves C_1 given by $y=\frac{5}{2}-\frac{13}{12}\,x-\frac{3}{4}\,x^2+\frac{1}{3}\,x^3$ on $-2\le x\le 2$ and C_2, which is the line segment from $(-2,-1)$ to $(2,0)$.

(a) Use *Maple* to plot the vector field and the curves C_1 and C_2 and determine if the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ is positive, negative or zero on each of C_1 and C_2.

(b) Is $\mathbf{F}$ a conservative vector field? How does your examination of the vector field and curves in (a) illustrate this? How can you determine this from the symbolic definition of the vector field?

(c) Use *Maple* to find the values of the two line integrals $\int_{C_1}\mathbf{F}\cdot d\mathbf{r}$ and $\int_{C_2}\mathbf{F}\cdot d\mathbf{r}$. Explain how this confirms your conclusion in (b).