

**Math 215**  
**Homework Set 9: §§17.6–17.8**  
**Fall 2009**

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 6th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

- 17.6.1: Problems #13–18 from §17.6.
- 17.6.2: Find a parametric representation for the part of the surface  $x^2 + y + 2z^2 = 4$  that lies to the right of  $y = 0$ . Then set up, but do not evaluate, an integral that gives the surface area of this part of the surface.
- 17.6.3: Sketch the sphere  $x^2 + y^2 + z^2 = a^2$  and the cylinder  $x^2 + y^2 = ax$ . Then set up an integral giving the area on the sphere and inside the cylinder. Evaluate your integral to find this surface area.
- 17.7.1: Consider the closed volume  $V$  contained by the cylinder  $y^2 + z^2 = 9$  and the planes  $x = 0$  and  $x + y = 5$ . Let the surface  $S$  be the boundary of this region. Note that this boundary consists of three parts.
- Sketch the volume  $V$  and surface  $S$ .
  - Find a parametric representation for each of the three parts of the boundary  $S$ .
  - Find the surface area element  $dS$  for each part.
  - Evaluate the surface integral  $\iint_S xz \, dS$  for this surface.
- 17.7.2: Consider the surface  $S$  consisting of the part of the cone  $z = \sqrt{x^2 + y^2}$  below the plane  $z = 1$  and lying in the first octant, with downward orientation.
- Sketch the surface  $S$ .
  - Find a parametric representation for the surface.
  - Find the vector area element  $d\mathbf{S}$  for  $S$ .
  - If  $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z^4\mathbf{k}$ , find the flux  $\iint_S \mathbf{F} \cdot d\mathbf{S}$  downward through the surface.
- 17.8.1: Consider the vector field  $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + xyz\mathbf{k}$  and the surface  $S$  which is the part of the plane  $2x + y + z = 2$  that lies in the first octant, oriented upward.
- Without making any calculations, is the flux through the surface positive or negative? Why?
  - Find the vector area element  $d\mathbf{S}$  for the surface, and calculate  $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$ .
  - What is the boundary  $\partial S$  of the surface? Set up and evaluate the line integral  $\oint_C \mathbf{F} \cdot d\mathbf{r}$  on  $\partial S$ .
  - Note that Stokes' Theorem holds. Which integral was easier to find?
- 17.8.2: Problem #16 from §17.8.
- M.7: *Maple* problem 7. Consider the surface  $S$  given by  $z = xy$ , for  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ .
- Find a parametric representation  $\mathbf{r}(t)$  for this surface, and then calculate the scalar and vector surface area elements  $dS$  and  $d\mathbf{S}$ .

- (b) Plot the surface using *Maple*. Will the value of the integral  $\iint_S xyz \, dS$  be positive, negative, or zero for this surface  $S$ ? Why? Will it be large or small in magnitude?
- (c) Use *Maple* to find an approximate numerical value for the integral  $\iint_S xyz \, dS$ .
- (d) Use *Maple* to plot the vector field  $\mathbf{F} = \sin(xyz) \mathbf{i} + x^2y \mathbf{j} + z^2e^{x/5} \mathbf{k}$  and the surface  $S$  together. Will  $\iint_S \mathbf{F} \cdot d\mathbf{S}$  be positive, negative or zero? Why? Will it be large or small in magnitude?
- (e) Use *Maple* to find an approximate numerical value for the integral  $\iint_S \mathbf{F} \cdot d\mathbf{S}$ .