Math 215
Homework Set 10: §§17.8–17.9
Fall 2009

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 6th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

17.8.3: Consider the curve C given by $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$ for $0 \le t \le 2\pi$. Using *Maple*, generate a graph of C. Generate a surface plot of the surface $z = 2xy$ and show the two together. Explain what this tells you. Then find (by hand, without using *Maple*)

$$\int_C (y + \sin x)\,dx + (z^2 + \cos y)\,dy + x^3\,dz.$$

(Hint: you may want to use Stokes' Theorem.)

17.9.1: Let E be the solid bounded by the paraboloid $z = 4 - x^2 - y^2$, the xy-, xz- and yz-planes.

(a) Sketch this region. Identify the four parts of the surface S bounding E and find parameterizations for each.

(b) Let $\mathbf{F}(x, y, z) = x^2\,\mathbf{i} + xy\,\mathbf{j} + z\,\mathbf{k}$. Find the flux outward through the bounding surface S.

(c) Finally, find $\iint_S \mathbf{F} \cdot d\mathbf{S}$ by using the Divergence Theorem.

17.9.2: Problem #18 from §17.9.

17.9.3: Consider the vector field $\mathbf{F}(x, y) = \langle x^2, xy^2 \rangle$.

(a) Sketch the vector field on the domain $[-3, 3] \times [-3, 3]$. Explain where you think $\operatorname{div} \mathbf{F}$ is positive and where it is negative, and why.

(b) Find $\operatorname{div} \mathbf{F}$ and determine where $\operatorname{div} \mathbf{F}$ is positive and negative; compare this with your analysis in (a).

M.8: *Maple* problem 8. Consider the curve C given by the intersection of the hyperbolic parabaloid $z = x^2 - y^2$ with the cylinder $x^2 + y^2 = 1$.

(a) Find parametric representations of the cylinder and the curve C, and use *Maple* to plot both surfaces and their intersection.

(b) If $\mathbf{F} = \langle x^2 y, x^3, xy \rangle$, set up the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$. Then evaluate it, using *Maple* if you like.

(c) For this same $\mathbf{F}$, find $\operatorname{curl} \mathbf{F}$ and set up the integral $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where S is the surface $z = x^2 - y^2$ bounded by the curve C. Evaluate the integral, using *Maple* if you like.

By Stokes' Theorem, the integrals in (b) and (c) should be equal. (So if you found that they aren't, you should go back and check your work!)