Math 215
Homework Set 10: §§16.8–16.9
Winter 2012

Most of the following problems are modified versions of the problems from your text book, *Multivariable Calculus*, 7th ed., by James Stewart. Your solution to each problem should be complete, show all work, and be written in complete sentences where appropriate. For *Maple* problems, include a print-out that shows all of the work and graphs that you generated in *Maple* to solve the problem, in addition to any work you may have done by hand.

16.8.1: Consider the vector field $\mathbf{F}(x, y, z) = -yz\,\mathbf{i} + y\,\mathbf{j} + 3x\,\mathbf{k}$ and the surface S which is the part of the plane $2x + y + z = 2$ that lies in the first octant, oriented upward.

(a) Without making any calculations, is the flux through the surface positive or negative? Why?

(b) Find the vector area element $d\mathbf{S}$ for the surface, and calculate $\iint_S \operatorname{curl}\mathbf{F} \cdot d\mathbf{S}$.

(c) What is the boundary ∂S of the surface? Set up and evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ on ∂S.

(d) Note that Stokes' Theorem holds. Which integral was easier to find?

16.8.2: Consider the 3-d force field

$$\mathbf{F}(x, y, z) = \langle z^2, 2xy, 4y^2 \rangle.$$

If a particle moves from the origin along straight line segments to the points $(1, 0, 0)$, $(1, 2, 1)$, $(0, 2, 1)$, and then back to the origin (in that order), how much work is done? (*Hint: Use Stokes' Theorem*).

16.8.3: Consider the curve C given by $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$ for $0 \le t \le 2\pi$. Using *Maple*, generate a graph of C. Generate a surface plot of the surface $z = 2xy$ and show the two together. Explain what this tells you. Then find (by hand, without using *Maple*)

$$\int_C (y + \sin x)\,dx + (z^2 + \cos y)\,dy + x^3\,dz.$$

(Hint: you may want to use Stokes' Theorem.)

16.9.1: Let E be the solid bounded by the sphere $x^2 + y^2 + z^2 = 16$, the xy-, xz- and yz-planes, lying in the first octant.

(a) Sketch this region. Identify the four parts of the surface S bounding E and find parameterizations for each.

(b) Let $\mathbf{F}(x, y, z) = z\,\mathbf{i} + y\,\mathbf{j} + x\,\mathbf{k}$. Find the flux outward through the bounding surface S.

(c) Finally, find $\iint_S \mathbf{F} \cdot d\mathbf{S}$ by using the Divergence Theorem.

16.9.2: Consider the vector field $\mathbf{F}(x, y, z) = z^{z^2} \arcsin(y/z)\,\mathbf{i} + (2x/z)\,\mathbf{j} + z\,\mathbf{k}$. Find the flux of $\mathbf{F}$ across the part of the paraboloid $x^2 + y^2 + z = 2$ that lies above the plane $z = 1$, oriented upwards.

16.9.3: Consider the vector field $\mathbf{F}(x, y) = \langle xy, x + y^2 \rangle$.

(a) Sketch the vector field on the domain $[-3, 3] \times [-3, 3]$. Explain where you think $\operatorname{div}\mathbf{F}$ is positive and where it is negative, and why.

(b) Find $\operatorname{div}\mathbf{F}$ and determine where $\operatorname{div}\mathbf{F}$ is positive and negative; compare this with your analysis in (a).

M.7: *Maple* problem 7. Consider the curve C given by the intersection of the hyperbolic parabaloid $z = x^2 - y^2$ with the cylinder $x^2 + y^2 = 4$.

(a) Find parametric representations of the cylinder and the curve C, and use *Maple* to plot both surfaces and their intersection.

(b) If $\mathbf{F} = \langle x^2 y, \frac{x^3}{3}, x \rangle$, set up the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$. Then evaluate it, using *Maple* if you like.

(c) For this same $\mathbf{F}$, find $\operatorname{curl} \mathbf{F}$ and set up the integral $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where S is the part of the surface $z = x^2 - y^2$ which is bounded by the curve C. Evaluate the integral, using *Maple* if you like.

By Stokes' Theorem, the integrals in (b) and (c) should be equal. (So if you found that they aren't, you should go back and check your work!)