1. Consider a cube with sides of length 2: see the picture. Let $A$ and $B$ be the vertices indicated in the picture. Let $Q$ be the point which is in the middle of an edge of the box (equal distance from the two end points of the edge) as indicated in the picture.

(a) (6 points) What is the angle between the line segment $QA$ and the line segment $QB$? (You can leave it as $\cos^{-1}$ of a real number.)

(b) (4 points) What is the area of the triangle formed by $A$, $B$ and $Q$?

2. (10 points) Consider the surface given by the equation $x^2 + y^2 + z^2 = 6$ and another surface given by the equation $z = x^2 + y^2$. The intersection of these two surfaces forms a curve. Find the length of this curve.

3. (10 points) Two objects travel through space along two different curves. The trajectory of object $A$ is given by the function $\mathbf{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle$, $t \geq 0$, and the trajectory of object $B$ is given by the function $\mathbf{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle$, $t \geq 0$.

(a) Do the objects collide? (Show your work in detail.)

(b) If your answer to (a) is yes, determine which object was traveling at a faster speed at the collision. On the other hand, if your answer to (a) is no, determine whether the curves represented by the trajectories intersect or not.

4. (10 points) Find an equation of the plane that passes through the point $(-1, 2, 1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

5. (10 points) Find an equation of the tangent plane to the surface given by the equation $x^2 + y^2 - z^2 = 12$ at point $(2, 3, 1)$. 
6. (No partial credit) Consider the following level curves. Answer your questions but there is no need for explanation.

(1) (3 points) Which picture represents the level curves of \( f(x, y) = \sqrt{x^2 + y^2} \)?

(2) (3 points) Which picture represents the level curves of \( f(x, y) = x^2 - y^2 \)?

(3) (4 points) The picture (i) is a contour of a function among the following choices. Circle the correct function.

\[
\begin{align*}
(A) \ \sin(x)\sin(y) & \quad (B) \ \sin(x) + \sin(y) & \quad (C) \ \sin(xy) & \quad (D) \ \sin(x + y) \\
(E) \ \frac{1}{x^2 + y^2 + 1} & \quad (F) \ \frac{x + y}{x^2 + y^2 + 1} & \quad (G) \ \frac{xy}{x^2 + y^2 + 1} & \quad (H) \ x^2 + y^2 + 1
\end{align*}
\]
7. (10 points) There is a function $f(x, y)$ which is differentiable whose exact formula is not known. Suppose, however, that we know that the intersection of the surface $z = f(x, y)$ and the plane $x = 1$ is given by the curve $\vec{r}_1(t) = \langle 1, 1 + t + t^2, 1 - 2t \rangle$, and the intersection of the surface $z = f(x, y)$ and the plane $y = 1$ is given by the curve $\vec{r}_2(t) = \langle t^3, 1, 2 - t \rangle$. We also know that $f(1, 1) = 1$. Find the partial derivatives $f_x(1, 1)$ and $f_y(1, 1)$ evaluated at $(x, y) = (1, 1)$.

8. (5 points) (No partial credit) Let $u(x, t) = \sin(x - t)$. Which of the equations does this function satisfy? Circle one from (a)–(g). Show your work.

(a) $u_t = u$
(b) $u_t = u_x$
(c) $u_t u_x = 0$
(d) $u_{tx} = 0$
(e) $u_t = u_{xx}$
(f) $u_{xx} + u_{tt} = 0$
(g) $u_{xx} - u_{tt} = 0$

9. (10 points) Consider a right circular cylinder. The radius of the right circular cone is increasing at a rate of 2 cm/s while its height is decreasing at a rate of 3 cm/s. At what rate is the volume of the cylinder changing when the radius is 20 cm and the height is 30 cm?