Math 215 — First Midterm
October 10, 2019

First 3 Letters of Last Name: □ □ □ ❄ UM Id#: _______________________
Instructor: ______________________ Section: ______________________

1. **Do not open this exam until you are told to do so.**

2. This exam has 13 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

3. Do not separate the pages of this exam, other than the formula sheet at the end of the exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.

4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.

5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.

6. You may use no aids (e.g., calculators or notecards) on this exam.

7. **Turn off all cell phones**, remove all headphones, and **place any watch you are using on the desk in front of you**.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
<td></td>
</tr>
</tbody>
</table>
1. [15 points] The sides of the cube below have length six. The points $p$ and $q$ are at the midpoints of their respective edges. Let $T$ be the triangle with vertices $a$, $p$, and $q$.

![Diagram of a cube with vertices labeled a, p, and q]

a. [8 points] What is the area of $T$?

b. [7 points] If $\theta$ is the smallest angle of $T$, what is $\cos(\theta)$?
2. [15 points] Indicate if each of the following is true or false by circling the correct answer.

a. [3 points] For any two vectors $u$ and $v$, we have $|u + v| = |u| + |v|$.
   
   True  False

b. [3 points] For any two vectors $u$ and $v$, we have $|u \times v| = |v \times u|$.
   
   True  False

c. [3 points] If $q(t)$ is a differentiable vector valued function, then $\frac{d}{dt}|q(t)| = |q'(t)|$.
   
   True  False

d. [3 points] Different parameterizations of a space curve $C$ result in identical tangent vectors at a given point on the curve.
   
   True  False

e. [3 points] If $u$ and $v$ are two unit vectors in $\mathbb{R}^3$, then the curve parameterized by $r(t) = \cos(t)u + \sin(t)v$ is a circle.
   
   True  False
3. [15 points] Beginning at the center of a disk of diameter 50cm, a ladybug crawls along a radius at a speed of 4 cm/sec. Meanwhile, the disk is turning counterclockwise about its center at 1/3 revolutions per second. If we assume that the origin is at the center of the disk, the ladybug initially walks in the direction of the positive x-axis, and the disk is in the plane \( z = 0 \), then, until the ladybug reaches the edge of the disk, the position of the ladybug is given by

\[
\mathbf{\ell}(t) = (4t \cos(\omega t), 4t \sin(\omega t), 0)
\]

where \( \omega = \frac{2\pi}{3} \).

a. [2 points] At what time would the ladybug reach the edge of the disk?

b. [4 points] Express, as an integral, how far the ladybug will have travelled during the first six seconds of its journey. Do not evaluate the integral.
c. [4 points] Provide a sketch of the path the ladybug would travel.

![Sketch of the path](image)

d. [5 points] At time $t = 6$ the ladybug loses its footing and begins to slide across the disk in the direction of the tangent vector $\ell'(6)$. At the same instant, the ladybug begins to fly with acceleration described by $\mathbf{a}(T) = \langle 0, 0, T/10 \rangle$ and initial velocity $\ell'(6)$. Here $T$ is the time that has passed since the ladybug lost its footing. Describe the position of the ladybug for $T \geq 0$. 
4. [10 points] Match each of the nine sets of level curves below with the appropriate function.

- \( \sin(xy) \)
- \( \sin(x) \sin(y) \)
- \( \sin(x) + \sin(y) \)
- \( \cos(x)/(x^2 + y^2 + 1) \)
- \( \sin(x)/(x^2 + y^2 + 1) \)
- \( (x^2 + y^2)^{1/2} \)
- \( e^x \cos(y) \)
- \( (1 - x^2)(1 - y^2) \)
- \( \sin(x + y) \)
5. [10 points] Suppose \( g(x, y) = e^{y-x} \).

a. [4 points] Find an equation for the tangent plane to the surface given by the equation \( z = g(x, y) \) at the point \((3, 3, 1)\).

b. [4 points] Find the linearization \( L_g(x, y) \) of the function \( g(x, y) \) at the point \((3, 3)\).

c. [2 points] Use the linear approximation \( L_g(x, y) \) to estimate \( g(2.9, 3.1) \).
6. [15 points] Find or estimate, depending on the type of data provided, the partial derivative in the $x$ direction and the partial derivative in the $y$ direction at the point $(3, 2)$ for each of the following functions.

a. [5 points] For a function $f$ given by the formula $f(x, y) = x^2 e^{xy^2}$

b. [5 points] For a function $g$ described by the data in the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>25</td>
<td>28</td>
<td>31</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>36</td>
<td>40</td>
<td>45</td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
<td>47</td>
<td>54</td>
<td>62</td>
<td>67</td>
</tr>
</tbody>
</table>
c. [5 points] For a function $h$ with level curves as given below.
7. [10 points] The trajectories for two particles are given by $\mathbf{r}_1(t) = (t^2 - 2, t^2 - 1, t^2)$ and $\mathbf{r}_2(t) = (\cos(\pi t), \sin(\pi t), t)$ for $0 \leq t \leq 2$. Let $C_1$ and $C_2$ denote the corresponding space curves.

a. [4 points] Graph $C_1$ and $C_2$.

b. [4 points] Do $C_1$ and $C_2$ intersect? Justify your answer.
c. [2 points] Do the particles ever collide? Justify your answer.

8. [10 points] Suppose $P$ is a plane that contains the points $(0, 3, 0), (-3, 0, 0), \text{ and } (2, 6, 1)$. Find an equation that describes the sphere centered at the origin that is tangent to $P$. 
You may use this page for scratch work.
• \( \sin^2(x) + \cos^2(x) = 1 \), \( \cos(2x) = \cos^2(x) - \sin^2(x) \), \( \sin(2x) = 2 \sin(x) \cos(x) \)

• \( \sin^2(x) = \frac{1 - \cos(2x)}{2} \), \( \cos^2(x) = \frac{1 + \cos(2x)}{2} \)

• \( \cos(\pi/3) = 1/2 \), \( \sin(\pi/3) = \sqrt{3}/2 \), \( \cos(\pi/4) = \sqrt{2}/2 \), \( \sin(\pi/4) = \sqrt{2}/2 \), \( \cos(\pi/6) = \sqrt{3}/2 \), \( \sin(\pi/6) = 1/2 \), \( \cos(0) = 1 \), \( \sin(0) = 0 \).

• \( \frac{d}{dx} \sin(x) = \cos(x) \), \( \frac{d}{dx} \cos(x) = -\sin(x) \).

• Volume of the parallelepiped determined by the vectors \( \mathbf{v}_1 = (a, b, c) \), \( \mathbf{v}_2 = (d, e, f) \), and \( \mathbf{v}_3 = (g, h, i) \) is \( |\mathbf{v}_1 \cdot (\mathbf{v}_2 \times \mathbf{v}_3)| \) = absolute value of \[
\begin{vmatrix}
a & b & c \\
d & e & f \\
g & h & i \\
\end{vmatrix}
\]

• Distance from a point \( (a, b, c) \) to a plane \( Ax + By + Cz + D = 0 \) is \( \frac{|Aa + Bb + Cc + D|}{\sqrt{A^2 + B^2 + C^2}} \).

• The circumference of a circle of radius \( a \) is \( 2\pi a \).

• The area of a disk of radius \( a \) is \( \pi a^2 \).

• The volume of a right circular cylinder of radius \( a \) and height \( h \) is \( \pi a^2 h \).