Name: ____________________________________________

MATH 215

MIDTERM I

This Exam contains 5 problems. The problems are worth 12 points each. Each part of a problem counts equally. On problems 3, 4 and 5 you can get partial credit. Hence, explain yourself carefully on these problems.

NO CALCULATOR.

1 TWO-SIDED 3in. BY 5in. NOTECARD OK.
CHECK YOUR SECTION IN THE TABLE

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Problem 1. TRUE/FALSE QUESTIONS. NO PARTIAL CREDIT. CIRCLE TRUE OR FALSE. IF YOU THINK A STATEMENT DOESN’T MAKE SENSE, CIRCLE FALSE.

(a) \[ \frac{\partial}{\partial x} \int_{x^2}^{x^3} \frac{y}{t} \, dt = -\frac{y}{x} \quad \text{if } x > 0 \] TRUE / FALSE

**ANSWER: TRUE**

(b) \[ \int \frac{dx}{x^2 + x^5} = -\frac{1}{x} - \frac{1}{4x^2} + C \] TRUE / FALSE

**ANSWER: FALSE**

(c) The formula \( \vec{n} \cdot (\vec{r} - \vec{r}_0) = \langle 0, 0, 0 \rangle \) determines a plane. TRUE / FALSE

**ANSWER: FALSE**

(d) The expression \( \vec{r}(t) = \langle \sin(e^t), \cos(e^t) \rangle, t \in [0, \ln(1 + 2\pi)] \) parametrizes a circle. TRUE / FALSE

**ANSWER: TRUE**

(e) The cross product of \( \vec{a} \) and \( \vec{b} \) equals the area of the parallelogram determined by \( \vec{a} \) and \( \vec{b} \). TRUE / FALSE

**ANSWER: FALSE**

(f) The gradient of a function is perpendicular to the level sets of the function. TRUE / FALSE

**ANSWER: TRUE**

WORKSPACE:
Problem 2. NO PARTIAL CREDIT Consider the points $A(0, 2, 1), B(1, 1, 2), C(2, 3, 1)$.

(a) Find $\vec{AB}$ and $\vec{AC}$. NOTE: CHECK THIS CALCULATION BECAUSE IF YOU MAKE A MISTAKE YOU WILL LOSE A LOT OF POINTS ON THE FOLLOWING QUESTIONS.

(b) Find the COSINE of the angle between $\vec{AB}$ and $\vec{AC}$.

(c) Find the SINE of the angle between $\vec{AB}$ and $\vec{AC}$.

(d) Find an equation for the line through $A$ and $B$.

(e) Find a vector perpendicular to the plane through $A, B, C$.

(f) Find the intersection point of the plane $x + y + z = 1$ and the line $\vec{r}(t) = <t, t - 2, 2>$. NOTE: THIS PART DOES NOT USE THE POINTS A,B,C.

ANSWERS:

a: $\vec{AB} = <1, -1, 1>, \vec{AC} = <2, 1, 0>$

b: $\sqrt{\frac{1}{15}}$

c: $\sqrt{\frac{14}{15}}$

d: $\vec{r}(t) = <0, 2, 1> + t <1, -1, 1>$

e: $<-1, 2, 3>$

f: $(1/2, -3/2, 2)$

WORKSPACE:
WORKSPACE PROBLEM 2
Problem 3. YOU CAN EARN 0, 2 OR 4 POINTS ON EACH PART. A SCORE OF 2 POINTS WILL BE AWARDED ONLY IN THE CASE OF A SMALL MISTAKE.

(a) Find the domain of the vector function \( \mathbf{r}(t) = \langle \frac{1}{t}, 2 + t, \ln(1 - t^2) \rangle \).

(b) Evaluate the integral

\[ \int_{1}^{2} < \frac{1}{t}, t e^t, t e^{t^2} > dt \]

(c) Find \( f_{xy} \) if \( f(x, y) = x y e^{x^2 y} \).

ANSWERS:

a: \((-1, 0) \cup (0, 1)\)

b: \( < \ln 2, e^2, e^4/2 - e/2 > \)

c: \( e^{x^2 y}[1 + 2x^2 + x^2 y + 2x^4 y] \)

WORKSPACE:
WORKSPACE PROBLEM 3
Problem 4. IF YOUR ANSWER IS CORRECT, YOU GET FULL CREDIT. IF ANSWER IS INCORRECT, YOU CAN GET PARTIAL CREDIT IF YOU EXPLAIN YOURSELF CAREFULLY.

(a) Find an equation for the tangent plane to \( z = ye^{x^2y} \) at \( (1, 2, 2e^2) \).

(b) Find the gradient of \( f(x, y, z) = \cos\left(\frac{\pi xy}{z}\right) \) at \( (1, 1, 2) \).

(c) Find the rate of change of \( f(x, y) = \frac{y}{x} \) in the direction of a unit vector parallel to \( \vec{u} = <1, -1> \) at \( (2, 2) \).

(d) If \( z = f(x, y) = \arctan\left(\frac{x}{y}\right) \), \( x = s + t \), and \( y = t^2 \), find \( \frac{\partial z}{\partial t} \) at \( (s, t) = (1, -1) \).

ANSWERS:

a: \( z = 2e^2 + 4e^2(x - 1) + 3e^2(y - 1) \)
b: \( <-\pi/2, -\pi/2, \pi/4> \)
c: \(-1/\sqrt{2}\)
d: 1

WORKSPACE:
WORKSPACE PROBLEM 4
Problem 5. IF YOUR ANSWER IS CORRECT, YOU GET FULL CREDIT. IF YOUR ANSWER IS INCORRECT, YOU CAN GET PARTIAL CREDIT IF YOU EXPLAIN YOURSELF CAREFULLY.

(a) Find the local maxima, local minima and saddle points of the function \( f(x, y) = (x^2 + 1) \sin y \).

(b) Find the maximum and minimum values of the function \( f(x, y, z) = xyz \) subject to the constraint \( x^2 + y^2 + z^2 = 1 \).

ANSWERS:

a. Saddle points: \((0, (k + \frac{1}{2})\pi), k \) integer:

b: \( \pm 1/(3\sqrt{3}) \)

WORKSPACE:

a) \( f_x = 2x \sin y, f_y = (x^2 + 1) \cos y \). Critical points are \((0, (k + \frac{1}{2})\pi), f_{xx}f_{yy} - f_{xy}^2 = 2 \sin y(-(x^2 + 1)) \sin y - (2x \cos y)^2 = -2(\sin y)^2 \)

All critical points are saddles.

b)

\[
\begin{align*}
\nabla f &= \lambda \nabla g \\
yz &= \lambda(2x) \\
xz &= \lambda(2y) \\
xy &= \lambda(2z) \\
xyz &= 2\lambda x^2 = 2\lambda y^2 = 2\lambda z^2
\end{align*}
\]

If \( \lambda = 0 \) at most one coordinate can be different from 0. We get the following seven points where \( f = 0 : (0, 0, 0), (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1). \)

If \( \lambda \neq 0 \), we get \( x^2 = y^2 = z^2 = 1/3 \) so we get the points \((\pm \sqrt{1/3}, \pm \sqrt{1/3}, \pm \sqrt{1/3}) \) Hence the max value of \( f \) is \( \frac{1}{3\sqrt{3}} \) and the min value is \( -\frac{1}{3\sqrt{3}} \).
WORKSPACE PROBLEM 5
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