1. (10 points) The position of a particle is given by \( \vec{r}(t) = \left( \frac{1}{2}t^2, 5t, \frac{1}{2}t^2 + 2t \right) \) at time \( t \). At what time is the speed minimum?

\[
\vec{v} = \langle -t, 5, t+2 \rangle
\]

Speed = \( |\vec{v}| = \sqrt{t^2 + 5^2 + (t+2)^2} = \sqrt{2t^2 - 4t + 29} \)

This \( \bigstar \) is \( \text{min. when} \)

\[ f(t) = 2t^2 - 4t + 29 \] \( \text{is \( \text{min.} \) } \)

\[ \Rightarrow f' = 4t - 4 = 0 \quad \Rightarrow t = 1 \]

The time is \( t = 1 \)
2. (10 points) Let \( \vec{A} = 2\hat{i} + 3\hat{j} + \hat{k} \). Let \( P \) be the plane \( x + y + z = 7 \). Write the vector \( \vec{A} \) as a sum

\[
\vec{A} = \vec{B} + \vec{C}
\]

where \( \vec{B} \) is perpendicular to the plane \( P \) and \( \vec{C} \) is parallel to the plane \( P \).

\[
\vec{B} = \text{proj}_\vec{n} \vec{A} = \frac{\vec{A} \cdot \vec{n}}{1/|\vec{n}|^2} \vec{n} = \frac{6}{3} \langle 1, 1, 1 \rangle = \langle 2, 2, 2 \rangle
\]

\[
\vec{C} = \vec{A} - \vec{B} = \langle 0, 1, -1 \rangle
\]

\[
\vec{B} = \langle 2, 2, 2 \rangle
\]

\[
\vec{C} = \langle 0, 1, -1 \rangle
\]
3. (10 points) Consider the curve given by the equation $x = t$, $y = 2 \cos t$, and $z = 2 \sin t$. There is one point $A$ on this curve with $x = -1$ and one point $B$ on this curve with $x = 1$. Consider the part of the curve from point $A$ to point $B$. Find the length of it.

The curve is a helix.

The point $A$ corresponds to $t = -1$ and the point $B$ corresponds to $t = 1$.

$$
\text{length} = \int_{-1}^{1} \sqrt{(x')^2 + (y')^2 + (z')^2} \, dt
$$

$$
= \int_{-1}^{1} \sqrt{5} \, dt = 2\sqrt{5}
$$

The length is $2\sqrt{5}$. 
4. (10 points) Find the equation of the plane that contains the point \((-1, 1, -1)\) and the line \(x = t, y = 2t + 1, z = 3t + 2\).

\[ \vec{A} = \langle -1, 0, 3 \rangle \]
\[ \vec{B} = \langle 1, 2, 3 \rangle \]

\[ \vec{n} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 3 \\ 1 & 2 & 3 \end{vmatrix} = \langle -6, 0, 2 \rangle \]

\[-6(x - 0) + 0(y - 0) + 2(z - 2) = 0 \]
\[-3x + 2z - 4 = 0 \]

The equation of the plane is \[3x - z + 2 = 0\]
5. Consider the plane \( P : x - 2y + 2z = 5 \) and the line \( L : \mathbf{r}(t) = (2t - 4, 2t + 1, t + 1) \).

(a) (5 points) Prove that \( P \) and \( L \) are parallel to each other.

\[
\begin{align*}
\langle 1, -2, 2 \rangle \cdot \langle 2, 2, 1 \rangle \\
= 2 - 4 + 2 = 0.
\end{align*}
\]

(b) (5 points) Find the distance between \( P \) and \( L \).

Pick any pt on \( L \), say \((-4, 1, 1)\)

Distance between this point and \( P \)

\[
\frac{|-4 - 2(1) + 2(1) - 5|}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{9}{3} = 3
\]

The distance is \( 3 \).
6. (10 points; No partial credit) Consider the following level curves. Answer your questions but there is no need for explanation.

(1) (2 points) Which picture represents the level curves of \( f(x, y) = y^2 \)?

(2) (2 points) Which picture represents the level curves of \( f(x, y) = \sin(2x) \sin(2y) \)?

(3) (2 points) Which picture represents the level curves of \( f(x, y) = xy \)?

(4) (2 points) Which picture represents the level curves of \( f(x, y) = \sqrt{x^2 + y^2} \)?

(5) (2 points) Which picture represents the level curves of \( f(x, y) = (x^2 + y^2)^2 \)?

(Sol) (1) (b), (2) (c), (3) (h), (4) (e), (5) (d)
7. (10 points) Consider the function 

\[ f(x, y) = F(x^2 + y^2) \]

where \( F \) is a function of single variable such that 

\[ F(5) = 2, \quad F'(5) = -7, \quad F''(5) = 3, \quad F'''(5) = 6. \]

Find the rate of change of \( f(x, y) \) at \((x, y) = (1, 2)\) with respect to \( x \) when \( y \) is fixed.

\[
\frac{\partial f}{\partial x} \bigg|_{(1,2)} = \left( F' \right) \bigg|_{(1,2)} \bigg( 2x \bigg) \\
= F'(5) \times 2 \\
= -14
\]

The rate of change is -14.
8. (10 points) Consider two surfaces, \( S_1: z = x^3 + y^4 \) and \( S_2: z = x^2 y^2 \pm xy \). The intersection of these two surfaces is a curve \( C \) and \( C \) contains the point \((1, 1, 2)\). Find the tangent line to the curve \( C \) at point \((1, 1, 2)\).

Tangent planes to \( S_1 \) and \( S_2 \) have normal vectors

\[ \vec{n}_1 = \langle 3, 4, 1 \rangle, \quad \vec{n}_2 = \langle 3, 3, -1 \rangle \]

Tangent line to \( C \) is parallel to \( \vec{n}_1 \times \vec{n}_2 = \langle -1, 0, -3 \rangle \)

Thus,

\[
\begin{align*}
  x &= t - 1 \\
  y &= 1 \\
  z &= -3t + 2
\end{align*}
\]

The equation of the tangent line is \( x = t - 1, \ y = 1, \ z = -3t + 2 \)