Winter 2017, MATH 215 Calculus III, Exam 2

3/23/2017, 6:10-7:40pm (90 minutes)

- Your name: ________________________________

- Circle your section and write your Lab time:

<table>
<thead>
<tr>
<th>Section</th>
<th>Time</th>
<th>Professor</th>
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<th>Lab Time (e.g. Th 10-11)</th>
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<td>20</td>
<td>9–10</td>
<td>Sema Gunturkun</td>
<td>Alex Leaf</td>
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<td>30</td>
<td>10–11</td>
<td>Mattias Jonsson</td>
<td>Robert Cochrane</td>
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<td>11–12</td>
<td>Sumedha Ratnayake</td>
<td>Harry Lee</td>
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<td>12–1</td>
<td>Sumedha Ratnayake</td>
<td>Deshin Finlay</td>
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<td>1–2</td>
<td>Yueh-Ju Lin</td>
<td>Rebecca Sodervick</td>
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<td>70</td>
<td>2–3</td>
<td>Yueh-Ju Lin</td>
<td>Jacob Haley</td>
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Instructions:
- This examination booklet contains 7 problems.
- If you want extra space, write on the back.
- DO NOT remove any sheets or the staple from the exam booklet.
- The formula sheet is not collected back and not graded.
- This is a closed book exam. Electronic devices, calculators, and note-cards are not allowed.
- Show your work and explain clearly.
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<th>Problem</th>
<th>Your score</th>
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1. (10 points) Find the volume below the surface $z = x^4 + y^4$ and above the square in the $x$-$y$ plane with vertices at $(x, y) = (±1, 0), (0, ±1)$. The square is shown here:

Volume =
Extra space:
2. Consider the iterated triple integral

\[ \int_{0}^{1} \int_{y}^{1} \int_{0}^{y} f(x, y, z) \, dz \, dx \, dy. \]

In this integral, \( z \) is innermost, \( x \) is in the middle, and \( y \) is outermost.

(a) (5 points) Rewrite the integral with \( x \) innermost, \( y \) in the middle, and \( z \) outermost.

(b) (5 points) Rewrite the integral with \( z \) innermost, \( y \) in the middle, and \( x \) outermost.

Solution to part (a):

Solution to part (b):
Extra space:
3. Let \( f(x, y) = x^4 + y^4 + 4xy \).

(a) (5 points) Find three critical points of \( f(x, y) \).

(b) (5 points) Pick one of the three critical points and classify it as local minimum, local maximum, or saddle.

Three critical points of \( f(x, y) \):

Classification of the underlined critical point:
Extra space:
4. (10 points) Find a critical point (you do not need to classify it as a local maximum or minimum) of

\[ f(x, y, z) = -x \log x - 2y \log y - 3z \log z \]

subject to the constraint

\[ g(x, y, z) = x + 2y + 3z - 1 = 0. \]

Evaluate \( f \) at that point. Here \( \log \) is the natural logarithm, as usual, so that \( \frac{d \log x}{dx} = \frac{1}{x} \).

The critical point is:

Value of \( f(x, y, z) \) at critical point is:
5. (10 points) Find the two points at which the parabola \( y = \frac{x^2}{2} \) intersects the circle \( x^2 + y^2 = 8 \). If \( D \) is the region bounded by that parabola and circle (see below), evaluate the double integral:

\[
\int \int_D x^2 y \, dx \, dy.
\]

The region of integration \( D \) looks as follows:

Note: Once you have a numerical answer you do not need to simplify it to a fraction. In the textbook, the area element \( dx \, dy \) in the integral is given as \( dA \).

Two points of intersection:

Value of double integral:
Extra space:
6. Consider the integral
\[ \int \int_{D} \frac{dx \, dy}{(x^2 + y^2)^{1/2}}, \]
where \( D \) is the disc \((x - 1)^2 + y^2 \leq 1\).

(a) (4 points) Describe the region of integration in polar coordinates.

(b) (6 points) Evaluate the integral.

Region of integration in polar:

Value of integral:
Extra space:
7. (10 points) Sketch the sector of the unit disc bounded by the lines $x = y$, $y = 0$, and the circle $x^2 + y^2 = 1$ in the first quadrant of the $x$-$y$ plane. Assuming constant density, find the $x$ and $y$ coordinates of the center of mass.

$x$-coord of center of mass:

$y$-coord of center of mass:
Extra space: