Your PRINTED name is: ________________________

Please circle your section and write down your lab time:

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Instructions

This is a 120 minutes closed book exam. No notes or calculators are allowed.
This examination booklet contains 7 problems, on 16 sheets of paper including this front cover. Check this before you start.
The next to last page is a list of formulas which you might find useful. The last page is blank and is to be used as scratch paper. Feel free to tear these last two pages away from the rest of the exam, but be careful to tear only those pages.
SHOW ALL YOUR WORK. Make sure your final answer is clearly written in the corresponding answer box.
DO NOT CHEAT! IF YOU CHEAT, YOU FAIL!
Grading

1
__________/30
2
__________/10
3
__________/20
4
__________/10
5
__________/20
6
__________/20
7
__________/10

Total:

/120
1 ((a)-(j): 30 pts.) No justification is required. No partial credit for this problem.

(a) [True or False] If \( \vec{a} \times \vec{b} = \vec{a} \times \vec{c} \), then either \( \vec{a} = \vec{0} \) or \( \vec{b} = \vec{c} \).

\[ \text{Circle your answer:} \quad \text{True} \quad \text{False} \]

(b) [True or False] The integral \( \int_{0}^{2\pi} \int_{0}^{4} \int_{0}^{r} dz \, dr \, d\theta \) represents the volume enclosed by the cone \( z = \sqrt{x^2 + y^2} \) and the plane \( z = 4 \).

\[ \text{Circle your answer:} \quad \text{True} \quad \text{False} \]

(c) [True or False] There is no smooth vector field \( \vec{F} \) on \( \mathbb{R}^3 \) such that \( \text{curl}(\vec{F}) = (x, 2y, 3z) \).

\[ \text{Circle your answer:} \quad \text{True} \quad \text{False} \]

Your workspace for (a)-(c) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
(d) [Fill in the blank] The straight line \( \vec{r}(t) = \langle t, t, t \rangle \) intersect the surface 
\[ 4z = x^2 + y^2 + 2 \] at the point ________________________.

(e) [Fill in the blank] The length of the curve 
\[ \vec{r}(t) = \langle 2 \cos t, 2 \sin t, \sqrt{5} \rangle, \quad 0 \leq t \leq 2\pi \]
is ________________________.

(f) [Fill in the blank] If \( f(x, y, x) = \sin(3x - yz) \), where \( x = e^{t-1}, y = t^3, z = t - 2 \), then \( \frac{\partial f}{\partial t}(1) \) equals ________________________.

(g) [Fill in blank] The portion of the surface \( z = \cos(x^2 + y^2) \) that lies inside the cylinder \( x^2 + y^2 = 4 \) can be represented by the parametric equation 
\( \vec{r}(u, v) = \) ________________________, where the domain of the parameter \((u, v)\) is given by the inequality ________________________.

Your workspace for (d)-(g) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
(h) [Multiple choice] How many critical points does the function \( f(x, y) = x^3 + y^3 - 6xy \) have?

Circle your answer: A: 0    B: 1    C: 2    D: 3

(i) [Multiple choice] Find \( a \) and \( b \) so that

\[
\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^3 (x + yz) \, dz \, dy \, dx = \int_0^a \int_0^b (x + yz) \, dx \, dy \, dz.
\]

Circle your answer: A: \( a = \sqrt{4 - z^2}, \ b = 2 \)    B: \( a = 2, b = x \)

C: \( a = 2, b = \sqrt{4 - y^2} \)    D: \( a = z, b = 2 \)

(j) [Multiple choice] How many of the following integrals calculate the area of a finite planar domain \( D \) with smooth boundary?

I: \( \iint_D dA \)    II: \( \int_{\partial D} xdy \)    III: \( \frac{1}{4} \int_{\partial D} 3xdy - ydx \).

Circle your answer: A: 0    B: 1    C: 2    D: 3

Your workspace for (h)-(j) %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
2 (10 pts.) Use Lagrange multipliers to find the maximum and the minimum values of 
\( f(x, y) = 10x^2 + 6y^2 \) subject to the constraint \( x^2 + 2y^2 - 14y = 0 \).

**YOUR ANSWER:**

The maximum value of \( f \) is

The minimum value of \( f \) is
This problem is about the “height” of points on the upper hemisphere

\[ x^2 + y^2 + z^2 = 1, \quad z \geq 0, \]

viewed as a function on its “shadow” (the unit disk in \( xy \) plane) in part (a), and viewed as a function on the hemisphere itself in part (b).

(a) Find the average of the function \( f(x, y) = \sqrt{1 - x^2 - y^2} \) over the disk \( x^2 + y^2 \leq 1 \) in the plane.

YOUR ANSWER: The average of \( f \) is

\[
\text{The average of } f \text{ is }
\]

\[
\text{YOUR ANSWER: The average of } f \text{ is }
\]
(b) Find the average of the function $F(x, y, z) = \sqrt{1 - x^2 - y^2}$ with respect to the surface area on this hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$.

\begin{center}
\text{YOUR ANSWER:} \hspace{1cm} \text{The average of } F \text{ is}
\end{center}
4 (10 pts.) Evaluate the integral
\[ \int_C (5y + \sqrt{x^{10} - 2\cos x + e^{-x}})dx + (8xy - \cos(e^{2y-2}))dy, \]
where $C$ is the parallelogram that consists of line segments from $(0,0)$ to $(2,6)$, then from $(2,6)$ to $(5,6)$, then from $(5,6)$ to $(3,0)$, and then from $(3,0)$ to $(0,0)$. 
If the vector field \( \vec{F} = \langle ay^2, 2xy + 2yz, by^2 + z^2 \rangle \) is a gradient field, then

(a) What must the values of \( a \) and \( b \) be?

\[
\begin{align*}
    a &= \\
    b &= 
\end{align*}
\]

[Use the value of \( a \) and \( b \) you get to answer the following problems]

(b) Find a potential function of \( \vec{F} \).
(c) Find the integral $\int_C \vec{F} \cdot d\vec{r}$, where the curve $C$ is parametrized by $x = e^t - te^t$, $y = 2t^2$, $z = 3t$, $0 \leq t \leq 1$.

\[
\int_C \vec{F} \cdot d\vec{r} =
\]

**YOUR ANSWER:**

(d) Give an equation of the surface $S$ that contains all points $P$ so that $\int_O^P \vec{F} \cdot d\vec{r} = 1$, where $O = (0, 0, 0)$ is the origin.

The surface equation is

**YOUR ANSWER:**
Let $\vec{F} = (x, y, 0)$, and let $E$ be the solid region bounded by the surface $z = 1 - x^2 - y^2$ and the $xy$ plane. Let $S$ be the boundary of $E$ (both parts) oriented outward.

(a) Calculate the flux of $\vec{F}$ across $S$ by using the divergence theorem.

YOUR ANSWER: The flux of $\vec{F}$ across $S$ is
(b) Recalculate the flux of $\vec{F}$ across $S$ by direct calculation as the sum of two surface integrals.

YOUR ANSWER:

The flux of $\vec{F}$ across $S$ is
Let $S$ be the surface $z = 8 - 2x^2 - 2y^2$, $z \geq 0$, oriented upward. Let

$$
\vec{F} = \langle x + y \cos z, 2xe^z - y, x^3y^2 - z^4 \rangle.
$$

Evaluate the integral $\iint_S \text{curl}(\vec{F}) \cdot d\vec{S}$.

(Hint: You might want to use a theorem.)

**YOUR ANSWER:**

$$
\iint_S \text{curl}(\vec{F}) \cdot d\vec{S} = 14
$$
Formulae Sheet

- Dot product $\langle a, b, c \rangle \cdot \langle d, e, f \rangle = ad + be + cf$.

- Cross product $\langle a, b, c \rangle \times \langle d, e, f \rangle = \langle bf - ce, cd - af, ae - bd \rangle$.

- Vector projection $\text{proj}_a \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \vec{a}$, scalar projection $\text{comp}_a \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$.

- Distance from a point $(a, b, c)$ to a plane $Ax + By + Cz + D = 0$ is $\frac{|Aa + Bb + Cc + D|}{\sqrt{A^2 + B^2 + C^2}}$.

- Length of curve $\vec{r}(t)$ from $\vec{r}(a)$ to $\vec{r}(b)$ is $\int_a^b |\vec{r}'(t)| \, dt$.

- Unit tangent $\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$, unit normal $\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$, binormal $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$.

- Second derivative test: $D = f_{xx}f_{yy} - (f_{xy})^2$.

- Polar coordinates: $x = r \cos \theta, y = r \sin \theta$.

- Spherical coordinates: $x = \rho \sin \phi \cos \theta, y = \rho \sin \phi \sin \theta, z = \rho \cos \phi$.

- The extra factor from Cartesian to polar coordinates is $r$.

- The extra factor from Cartesian to spherical coordinates is $\rho^2 \sin \phi$.

- The center of mass of a 2-d object [with shape $D$, total mass $m$ and density $\rho(x, y)$] is $(\bar{x}, \bar{y})$, where $\bar{x} = \frac{1}{m} \iint_D x \rho(x, y) \, dA, \bar{y} = \frac{1}{m} \iint_D y \rho(x, y) \, dA$.

- The volume of a ball of radius $r$ is $\frac{4\pi r^3}{3}$.

- The surface area of a sphere of radius $r$ is $4\pi r^2$.

- $\sin(2x) = 2 \sin x \cos x, \cos(2x) = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$.

- $\text{curl} \vec{F} = \nabla \times \vec{F}, \text{div} \vec{F} = \nabla \cdot \vec{F}$, where $\nabla = \langle \partial_x, \partial_y, \partial_z \rangle$.

- Green’s theorem: $\int_{\partial D} P \, dx + Q \, dy = \iint_D (Q_x - P_y) \, dA$.

- Stokes’ theorem: $\int_{\partial S} \vec{F} \cdot d\vec{r} = \iint_S (\text{curl} \vec{F}) \cdot d\vec{S}$.

- Divergence theorem: $\iiint_E (\text{div} \vec{F}) \, dV = \iint_{\partial E} \vec{F} \cdot d\vec{S}$.

- The area of parametrized surface $\vec{r} = \vec{r}(u, v)$ is $\iint_D |\vec{r}_u \times \vec{r}_v| \, dA$. 
This page is intended for use as scratch paper.