Please do all your work in this booklet and show all the steps. Make sure to box your final answer.

You do not need to simplify your answer, but we expect you to know and simplify some basic expressions, like $\sqrt{4}$ or $\cos \pi$.

Calculators and note-cards are not allowed.

Some useful trigonometric identities:

\[
\begin{align*}
\sin^2 \theta + \cos^2 \theta &= 1 \\
\sin^2 \theta &= \frac{1 - \cos 2\theta}{2} \\
\cos^2 \theta &= \frac{1 + \cos 2\theta}{2}
\end{align*}
\]

\[
\begin{align*}
\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\
\sin 2\theta &= 2 \sin \theta \cos \theta
\end{align*}
\]

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Problem 1. (10 pts.)

(a – 5 pts.) Find the distance from the point \( P(1, -3, 2) \) to the plane \( \Pi \) given by the equation \( y - z - 1 = z - x + 1 \).

(b – 5 pts.) Find an equation of the plane containing the line \( \mathbf{r}(t) = \langle 2 - t, t, 7t + 1 \rangle \) and the point \( A(-1, 1, 3) \).
Problem 2. (15 pts.) Suppose that you have just reached the final level of the exciting computer game “Horrible Monsters Galore”. You find yourself in a large hall at the point $(-1, 1)$, and a measure of your danger is given by $f(x, y) = y^2 - xy + \sin \left( \frac{\pi x}{2} \right)$ (larger values of $f$ indicate greater danger).

(a – 5 pts.) You decide to go directly north. Is it a good idea or a bad idea (briefly explain your reasoning)?

(b – 5 pts.) How does the danger change if you go in the direction of the vector $v = \langle 2, -1 \rangle$ instead (please give an actual value for the rate of change)?

(c – 5 pts.) The graph below shows the view of the hall together with the level curves and gradient vectors of the danger function. What would be a safe place to be at (i.e., find a spot of least danger). Where would a monster (monsters?) most likely be? Mark these points on the graph and explain very briefly.
Problem 3. (10 pts.) Find absolute minimum and absolute maximum values of the function $f(x, y) = xy$ on the region $D = \{(x, y) \mid 9x^2 + 16y^2 \leq 144\}$. Where are these values attained?
Problem 4. (10 pts.) Consider the following iterated triple integral:

\[ \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{\frac{1}{2} \sqrt{x^2+y^2}}^{1+x^2+y^2} 1 \, dz \, dy \, dx. \]

Carefully sketch the region of integration. Evaluate the integral and explain what the resulting number represents.
Problem 5. (10 pts.)

Consider the surface $S$ in the shape of the “african drum” (see the picture on the right). The top and bottom parts are flat disks and the side is a part of the surface given by the equation $x^2 + y^2 = z^2 + 1$, $-2 \leq z \leq 2$. Find the mass of this surface if the density function for the top and bottom parts is $5$ gr/cm$^2$ and the density of the side is $3|z|$ gr/cm$^2$. 
Problem 6. (5 pts.)
Consider the vector field \( \mathbf{F} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j} \), where \( P = \sin(x^2) - y \) and \( Q = x^2 - e^{y+1} \). Use Green's Theorem to find the circulation of the field \( \mathbf{F} \) along the boundary \( C \) of the region \( D \) shown on the picture on the right.

Problem 7. (5 pts.) Find the flux of a vector field \( \mathbf{F} = (x^2 - y^2)\mathbf{i} + 2xz\mathbf{j} + (z^2 - 2xz)\mathbf{k} \) across the surface \( S \) of the sphere of radius 3 centered at the origin (with the outward orientation).
Problem 8. (15 pts.)

Compute the circulation of the vector field \( \mathbf{F} = -y \mathbf{i} + (x - z) \mathbf{j} + xz \mathbf{k} \) along the boundary of the part of the sphere of radius 4 (centered at the origin) below the plane \( z = 2 \), if the boundary curve \( C \) is oriented counterclockwise if viewed from above (see the picture). Do the calculation in two different ways: directly and by using the Stokes’ Theorem.

(a – 7 pts.) Direct calculation:

\[ \int_C \mathbf{F} \cdot d\mathbf{r} \]

(b – 8 pts.) Using Stokes’ Theorem:

\[ \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} \]
Problem 9. (10 pts.) Consider the surface $S$ which is a part of the plane $3x + y - z = 6$ contained inside the cylinder $x^2 + y^2 = 1$ oriented upward.

(a – 5 pts.) Parametrize $S$. Make sure to explicitly describe or sketch the parametrization domain $D$.

(b – 5 pts.) Find the flux of the vector field $\mathbf{F}(x, y, z) = xy\mathbf{i} + zy\mathbf{j} - x^2\mathbf{k}$ across $S$. 
Problem 10. (10 pts.) Consider the plots of two vector fields: $F$ (left) and $G$ (right).

(a – 2 pts.) On either of the plots carefully mark a point at which the field has a non-zero divergence and indicate if it is positive or negative.

(b – 2 pts.) On either of the plots carefully mark a point at which the field has a non-zero curl and indicate if it is positive or negative.

(c – 2 pts.) On either of the plots sketch a closed trajectory such that the circulation of the vector field along it is non-zero and indicate if it is positive or negative.

(d – 2 pts.) On either of the plots give an example of two trajectories having the same starting and ending points such that the work of the field along these trajectories is different.

(e – 2 pts.) Comment on the possibility of each vector field being a gradient vector field.