READ THIS
This exam contains eight problems, worth a total of 100 points. The first four questions are multiple choice. Your answers for these four questions are to be entered in the table below. No partial credit will be given for the first four problems, so double check your work. Please do not cheat. The use of books, calculators, cell phones, computers, notes, cheat sheets, and all similar aids is strictly prohibited. Good luck.

SHOW YOUR WORK.

<table>
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<th>Problem</th>
<th>Indicate your answer</th>
<th>Points</th>
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<tr>
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Problem 1. \(15 = 5 + 10\) points

(a) (5 points) If \(\vec{u} = \langle 3, 1, 2 \rangle\) and \(\vec{v} = \langle -1, 2, 2 \rangle\), then \(\vec{u} \cdot \vec{v}\) equals

A. 1
B. \(-1\)
C. 3
D. 13
E. None of the above.

(b) (10 points) If \(\vec{u} = \langle 3, 1, 2 \rangle\) and \(\vec{v} = \langle -1, 2, 2 \rangle\), then \(\vec{u} \times \vec{v}\) equals

A. \(\langle -3, 2, 4 \rangle\)
B. \(\langle -2, -8, 7 \rangle\)
C. \(\langle -3, -2, 4 \rangle\)
D. \(\langle -2, 8, 7 \rangle\)
E. None of the above
Problem 2. (5 points) Let $P_1$ denote the plane described by the equation $x + 2y - z = 0$ and let $P_2$ denote the plane described by the equation $x + 2y - z = 16$. Note that $P_1$ and $P_2$ are parallel planes. Exactly one of the following statements is true; which one?

A. The points $(4, 2, 0)$ and $(7, -10, 13)$ both lie between $P_1$ and $P_2$.
B. Neither of the points $(4, 2, 0)$ or $(7, -10, 13)$ lies between $P_1$ and $P_2$.
C. The point $(4, 2, 0)$ lies between $P_1$ and $P_2$ while the point $(7, -10, 13)$ does not.
D. The point $(7, -10, 13)$ lies between $P_1$ and $P_2$ while the point $(4, 2, 0)$ does not.
E. None of the above statements are true.
Problem 3. (10 = 5 + 5 points)

(a) (5 points) The direction (expressed as a unit vector) of steepest ascent for the function \( f(x, y) = x^2y + y^2x + x \) at the point \((3, -1)\) is

A. \((-4, 3)\)
B. \((4, -3)\)
C. \((4/5, -3/5)\)
D. \((-4/5, 3/5)\)
E. None of the above.

(b) (5 points) The critical point \((-2/\sqrt{3}, 1/\sqrt{3})\) for the function \( f(x, y) = x^2y + y^2x + x \) is

A. a local minimum for \( f \).
B. a local maximum for \( f \).
C. a saddle point for \( f \).
D. All of the above.
E. None of the above.
Problem 4. (10 = 5 + 5 points)

(a) (5 points) Suppose \( C \) is the boundary of a domain \( D \) with \( C \) oriented as in the statement of Green’s theorem. We have that

\[
\oint_C (3x^2y^2 + e^{xy} + 1) \, dx + (e^y + xy^2 + 2yx^3) \, dy
\]

is equal to

A. \( \iint_D (-6x^2y^2 - ye^{xy} + e^y + 2xy + 2x^3) \, dA \)

B. \( \iint_D (y^2 - xe^{xy}) \, dA \)

C. \( \iint_D (-y^2 + xe^{xy}) \, dA \)

D. \( \iint_D (6x^2y^2 + ye^{xy} - e^y - 2xy - 2x^3) \, dA \)

E. None of the above.
(b) (5 points) Let $C$ be any curve from the point $(2, 3)$ to the point $(-1, 8)$. The integral

$$\int_C (2xy + y^2 + 1) \, dx + (x^2 + 2yx) \, dy$$

is equal to

A. 25  
B. $-25$  
C. 89  
D. $-89$  
E. The answer depends on $C$. 
Problem 5. (15 = 5 + 10 points) Suppose $S$ is that part of the cylinder of radius 9 centered about the $z$-axis which lies above the plane described by the equation $z = -17$, below the plane described by the equation $z = 17$, and in the half space defined by the equation $y \geq 0$.

(a) (5 points) Sketch $S$.

(b) (10 points) Evaluate

$$\int \int_S f \, dS$$

where $f(x, y, z) = xy^2z^2$. 
Continue your work for Problem 5 here.
Problem 6. (20 = 10 + 5 + 5 points)
(a) (10 points) Find an equation of the tangent plane at the point (18, 6, 12) to the parametric surface $S$ given by

$$r(u, v) = \langle u^2v, vu, uv^2 \rangle$$

where $0 \leq u \leq 5$ and $0 \leq v \leq 7$. 

(b) (5 points) Give a parametrization of the tangent line at the point (18, 6, 12) to the parametric curve \( C \) given by
\[
r(t) = \langle 2t^2, 2t, 4t \rangle
\]
where \( 0 \leq t \leq 5 \).

(c) (5 points) Show that the line found in part (b) belongs to the plane found in part (a). Also, in fifteen words or less, explain why the line found in part (b) belongs to the plane found in part (a).
Problem 7. (10 points) Suppose $C$ is the curve parametrized by $\mathbf{r}(t) = (3\cos(t), 3\sin(t), 2)$ for $0 \leq t \leq 2\pi$. Compute the line integral of $\mathbf{F}(x, y, z) = (-x^2yz, xy^2z, e^{xy})$ along the curve $C$. (Hint: Stokes’ Theorem may be useful.)
Continue your work for Problem 7 here.
Problem 8. (15 = 5 + 10 points) Let $P$ be the prism bounded below by the plane with equation $z = 0$, on the sides by the planes with equations $y = 0$, $y = 5$, and $x = 0$, and “above” by the plane with equation $z = 2 - x$.

(a) (5 points) Sketch $P$.

(b) (10 points) Let $S'$ be the boundary of $P$ and let $S$ be the surface obtained by removing the bottom face (i.e., the face in the $xy$-plane) from $S'$. We suppose that $S$ has “outward” orientation. Note that $S$ has four sides. Compute the flux across $S$ of the vector field $\mathbf{F}(x, y, z) = \langle x^3y, x^2y^2, x^2y(z + 1) \rangle$. (Hint: the divergence theorem may be useful.)
Continue your work for Problem 8 here.

END OF EXAM

Before leaving the examination room:
(1) Make sure you have transferred your answers for problems 0 – 3 to the chart on page one.
(2) Make sure you have placed your name and section number on page one.
(3) Make sure you turn in your exam to the proctor.