

Name: \_\_\_\_\_

Section/Time of lecture: \_\_\_\_\_

Professor/GSI: \_\_\_\_\_

## **MATH 216**

## **MIDTERM I**

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Each part of a problem counts equally. To get full score you need to carefully explain what you did. No calculators allowed.

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Problem	Points	Score
1	30	
2	30	
3	30	
4	60	
TOTAL	150	

**Problem 1.**

a) Verify that the function  $y(x) = \frac{1}{x} + \ln x$  satisfies

$$x^2 y'' + 2xy' - xy = -x \ln x$$

b) Solve  $\frac{dy}{dx} = \sin x(\cos x)^2 + \frac{1}{x^2}$ ,  $y(2\pi) = 1$ .

c) Sketch the slope field and solution curves for  $\frac{dy}{dx} = \sqrt{1-x^2}$  where defined. Include enough detail to be able to give a rough sketch of the solution curves with  $y(0) = 0$  and  $y(0) = 1$ .

**Problem 2.**

a) Solve  $\frac{dy}{dx} = x(1 - 2y)e^{2x}$ ,  $y(0) = 2$

b) Solve  $(x^2 + 1)y' = -3xy + x$ ,  $y(0) = 0$ .

c) The time rate of change of a population  $P$  is proportional to the cube root of  $P$ . At time  $t=0$ , the population is 1000 and the rate of increase is 100 at that time. What is the population at time  $t=10$ ?

**Problem 3.**

a) Let  $\frac{dx}{dt} = -5x + 2x^3$ . Find the equilibrium solutions. Which ones are stable?

b) Suppose that a motor boat is moving at 100 m/s when the motor quits. After 20 seconds, the boat has slowed to 10 m/s. Assume the resistance is proportional to its speed. Hence we have the differential equation for the speed  $v$  :

$$\frac{dv}{dt} = -kv.$$

How far will the boat coast?

c) Use Euler with stepsize .5 to estimate  $y(2)$  for  $\frac{dy}{dx} = x^2 + y^3, y(1) = 0$ .

**Problem 4.**

- a) Solve the equation  $y'' - 3y' - 10 = 0$  with initial conditions  $y(0) = 3, y'(0) = -1$ .
- b) Solve the equation  $5z^2 - 2z + 10 = 0$ .
- c) Solve  $9y'' + 6y' + 25y = 0, y(0) = 2, y'(0) = 1$