

Name: _____

Section/Time of lecture: _____

Professor/GSI: _____

MIDTERM II

Each part of a problem counts equally. To get full score you need to carefully explain what you did. No calculators allowed.

Problem	Points	Score
1	60	
2	60	
3	60	
4	60	
+	10	10
TOTAL	250	

Problem 1.

a) Find a particular solution of

$$y'' + y' + y = e^{2x}.$$

b) Find a particular solution of

$$y'' - 4y = e^{2x}.$$

c) Use the method of variation of parameters to find a particular solution of

$$y'' - y = xe^{2x}$$

d) A mass of 10 kg is attached to a spring which stretches 1m if you use a force of 5N. A force $F_0 = (\sin(\omega t))N$ is applied to the mass. For which value of ω will resonance occur? [There is no need to change units. There is no penalty for messing up on units.]

Problem 2.

a) Transform the system

$$\begin{aligned}x'' &= 2x' - y \\y'' &= 2y' - x\end{aligned}$$

into an equivalent system of first order differential equations:

$$\begin{aligned}x'_1 &= \\x'_2 &= \\x'_3 &= \\x'_4 &= \end{aligned}$$

b) Solve

$$\begin{aligned}\frac{dx}{dt} &= 2y, x(0) = 1 \\ \frac{dy}{dt} &= -3x, y(0) = 2\end{aligned}$$

c) Use the method of elimination to find the general solution of

$$\begin{aligned}\frac{dx}{dt} &= -2x + 3y, \\ \frac{dy}{dt} &= 2x - y\end{aligned}$$

d)

Consider the system:

$$\begin{aligned}x' &= 2x - y \\ y' &= 3x + y\end{aligned}$$

Estimate $x(2), y(2)$ using Eulers method with stepsize $h = 1$ if $x(0) = 1, y(0) = 2$.

Problem 3.

a) Calculate the matrix products AB and BA .

$$A = \begin{pmatrix} 1+i & 0 \\ 2 & i \end{pmatrix}$$

$$B = \begin{pmatrix} i & 2 \\ 0 & 3 \end{pmatrix}$$

b) Calculate the determinant of

$$\begin{pmatrix} 3+i & -i \\ 6 & 2 \end{pmatrix}$$

c) Find the eigenvalues and eigenvectors of

$$\begin{pmatrix} -1 & 2 \\ -9 & 5 \end{pmatrix}$$

d) Use the Eigenvalue Method to find the general solution of

$$\begin{aligned} \frac{dx}{dt} &= -x + 13y, \\ \frac{dy}{dt} &= -2x + y \end{aligned}$$

Problem 4.

a)

Find the general solution of

$$x'' = -3x + y$$

$$y'' = 2x - 2y$$

b)

$$\frac{dx}{dt} = 3x,$$

$$\frac{dy}{dt} = -2y$$

Draw a phase portrait and Direction field.

c)

$$\begin{aligned}\frac{dx}{dt} &= -5y, \\ \frac{dy}{dt} &= -2x\end{aligned}$$

Decide if $(0, 0)$ is stable.

d)

$$\begin{aligned}\frac{dx}{dt} &= x + x^2 - 2xy, \\ \frac{dy}{dt} &= y + 2y^2 - 3xy\end{aligned}$$

Find all critical points.