LAB 1: CANCER, SERIES, AND ODE SOLUTIONS, PART B

MATLAB commands we use in this lab are ode45 and an add-on function seriesapprox_w19, as well as plot to plot solutions and save to save variables for future use. These are (re)introduced below. The lab assignment follows this section.

1.1. ode45. Finds a numerical approximation to a differential equation.
   >> [tsol,ysol] = ode45( func_handle, [tmin,tmax], initial_v );

1.2. plot. Plot one vector against another.
   >> plot( t_values, y_values );

1.3. axis. Set the x and y domains of a plot:
   >> axis( [xmin xmax ymin ymax] );

1.4. hold. Plot later plots on the same axes (hold on), or replace the current plot (hold off):
   >> hold on;
   >> hold off;

1.5. title, xlabel, ylabel. Set the title, x- and y-axis labels for an existing plot:
   >> title( 'title text' );
   >> xlabel( 'x-axis label text' );
   >> ylabel( 'y-axis label text' );

1.6. legend. Set the legend for the plot:
   >> legend( 'label 1', 'label 2', 'label 3'...);

2. BACKGROUND

In this lab, we are studying the Gompertz equation, a first-order ordinary differential equation which models the growth of cancerous tumors,

\[
\frac{dy}{dt} = ry \ln(K/y).
\]

The constants \(r\) and \(K\) in this equation are positive, and we consider \(r = 0.1\) and \(K = 10\). The function \(y(t)\) gives the volume of the tumor at time \(t\). The initial condition, \(y(0) = y_0\), must be positive (that is, greater than zero), and we will in general take \(y(0) = 1\).

You will complete a lab report as described in section 6 with your partner. This is due at the beginning of the next lab period.
3. Exercise 1

In your written homework you found the exact solution to the Gompertz equation. Check that you and your partner agree on the solution. If you haven’t already, also find the exact solution to the $n = 1$ (linear) approximation to the differential equation.

Take $r = 0.1$, $K = 10$, and $y(0) = 1$. Plot the exact solution to the Gompertz equation, (1), with the exact solution to the linear approximation (about $K$—you found this in Exercise 3 of the Prelab) and a numerical solution (generated with ode45) of the quadratic ($n = 2$) approximation.

How do the different solutions differ? How and where are they similar? Compare with your work in Part A, Exercise 3; does your work here give you confidence in the accuracy of the approximations to solutions of a differential equation that are generated by ode45?

4. Exercise 2

For all of the preceding work we have taken $y(0) = 1$. Would you expect the approximations to the Gompertz equation by expanding it around $y = K$ to be good approximations when $y = 1$? (Your work from Part A, Exercise 3 may shed some light on this.)

Let’s consider some values close to the expansion point. Find the exact solution to the Gompertz equation and the linear approximation when $y(0) = 8$ (you may take $r = 0.1$ and $K = 10$ still). Find numerical approximations using ode45 to the approximations with $n = 2$ and $n = 3$. Plot all of these solutions together. Do the solutions to the approximate equations look similar to the exact solution? Do the different approximations behave as you expect?

5. Exercise 3

Finally, recall in the Prelab we looked at the expansion of the differential equation around the point $y = 1$. Check with your partner that you agree on the form of the expansion in this case. Then use ode45 to generate (approximate) solutions to the Gompertz equation and the $n = 1$, $n = 2$, and $n = 3$ Taylor approximations to the equation (you may need to pick carefully the time interval on which you are generating the solution) and plot them. How good are the approximations? What happens to the agreement between the solutions to the approximate equations and the solution to the Gompertz equation as time goes on? Be sure you can explain why this makes sense given what your work in Exercise 2 suggests about how well the solutions to the approximate equations give insight on the behavior of the original.

6. Lab Report

Imagine that you are a biomedical engineering consultant, and that you are responding to the following request. Your lab writeup should respond to this, and be collaboratively produced by you and your partner. Both of you will submit the writeup paper. Note that you will need to include figures from the
work that you did in the course of Parts A and B of the lab to produce a good
writeup\(^1\), and that you will need to include the equations and mathematical
work underlies your conclusions.

The researcher’s request is as follows:

Dear Consultant:

I am studying the development of cancer tumors, and need your assistance
in understanding the predictions and validity of the Gompertz model of cancer
tumor size. In particular, I need you to determine what the model predicts
for the behavior of the tumor in certain treatment regimes, and when different
approximations to the model may be appropriate (or not).

Specifically:

• If a treatment reduces the rate of tumor growth, will that have a sig-
nificant impact on the long-term outcome of the cancer?
• What is the predicted long-term behavior of the tumor, and would this
be altered if the initial tumor size was changed, e.g., by a surgical
intervention that removed most of the tumor?
• What type of behavior is predicted for the tumor by the simplified form
of the Gompertz model? Is a simplified form of the Gompertz model ade-
quate to predict the behavior of the tumor, and are there circumstances
in which the simplification would be significantly better or worse?
• If the model is simplified by assuming a small tumor size, what can (and
can not) be determined from the resulting simplified model?

I look forward to receiving your report. The technical requirements for the
report, dictated by our lab, are that it should have the following format:

I. Introduction: The introduction should summarize the purpose and
contents of your report in 3–6 complete sentences. You should include
the Gompertz equation and the other equations you consider, with a
note on how you analyze them, but otherwise keep the technical notation
to a minimum.

II. Body: The body of the report should answer the questions of interest,
by describing how the different parts of your analysis allow you to do
this. In particular, all of the following should appear as you frame your
answers.

   a. You should note what the equilibrium solutions are for the Gom-
   pertz equation, and what your analysis says about how the different
   parameters in the equation (\(r\) and \(K\)) affect the solution.
   b. You should explain how Taylor expansions are used to simplify the
   Gompertz equation, and when the simplifications are likely to be
good approximations to the original model.

\(^1\)How do you get figures from MATLAB? You can do this by exporting them: in the figure
window, select File \(\rightarrow\) Export Setup. You can accept the defaults, and click Export. Select a
file type, e.g., JPEG, give the export a name, and click Save.
c. And you should explain what the approximation obtained in Exercise 3 tells about solutions to the Gompertz equation, and the circumstances under which it might be useful.

III. Conclusion: Provide a short, several paragraph, summary of your results that ties together the work you have described in the body.

Finally, as a 2 or 3 sentence appendix, include a reference or “bibliography” that connects your work in math 216 to the analysis you have done in the report. For example, you may want to indicate the section(s) of the text, or lectures from class, that treated the mathematics that you include in your report.

REFERENCES