Instructions: Solve each of these problems. Your solution should be complete and written out in complete sentences.

1. Problem 6 in §2.5 of Brannan and Boyce (p.91 in the 3rd ed. text). Also complete part (c), below.
   (c) Suppose that the equation was instead $y' = y(\alpha - y^2)$. Repeat your analysis from part (a). Note that your answer will depend on whether $\alpha < 0$, $\alpha = 0$, or $\alpha > 0$. To see what is going on, read through the text about Bifurcation Points on p.92.

2. Problem 26 in §3.1 of Brannan and Boyce (p.129 in the 3rd ed. text). Also complete parts (a)–(c) below.
   (a) Suppose that we solve $Ax = 0$. How many solutions are there? Why? What are they?
   (b) Suppose that the matrix $A = \begin{pmatrix} 1 & \alpha \\ 5 & -3 \end{pmatrix}$. For what $\alpha$, if any, are there no solutions to $Ax = 0$?
   (c) Suppose that the matrix $A = \begin{pmatrix} 1 & \alpha \\ 5 & -3 \end{pmatrix}$. For what $\alpha$ are there an infinite number of solutions to $Ax = 0$? Why? What are the eigenvalues and eigenvectors of $A$ in this case? Which could you have predicted? Why?

3. Problem 17 in §3.2 of Brannan and Boyce (p.143 in the 3rd ed. text). For (b), use dirfieldsys2 and ode45 in Matlab to generate the graph. Also complete parts (d) and (e) below.
   (d) Let $x = x_c + u$ and $y = y_c + v$, where $x_c$ and $y_c$ give the critical point you found in (a). Plug these into the system and show that you obtain a homogeneous system $u' = Au$ for $u = (u \ v)^T$.
   (e) Solve the resulting homogeneous system for $u$ and $v$, and show that the solutions you obtain match the phase portrait that you generated in (b).

4. Consider the van der Pol equation from lab 2, $x'' + \mu(x^2 - 1)x' + x = 0$.
   (a) Find all critical points of the equation.
   (b) Write the equation as a system of two first-order differential equations and find the critical point of the system; show that this is the same as you obtained in (a).
   (c) Explain why the linearization of the system you obtained in (b) is the system given in equation (5) of the prelab.
   (d) Find the eigenvalues and eigenvectors of that system.