1. In lab 3 we consider the nonlinear system

\[ N' = \gamma(A - N(1 + P)), \quad P' = P(N - 1). \]

We continue our analysis of this system here.

(a) Find all critical points of this system.

(b) Move the origin of the system to \((1, A - 1)\) by making the substitution

\[ N = 1 + u, \quad P = (A - 1) + v \]

in the original system.

(c) If \(|u|\) and \(|v|\) are very small, your system in (b) simplifies to

\[ u' = -A\gamma u - \gamma v, \quad v' = (A - 1)u. \]

Show that eigenvalues of this system change as \(A\) increases through \(A = 1\), and explain why this suggests that the critical point is unstable for \(A < 1\) and stable for \(A > 1\).

2. Problem 32 in §4.2 of Brannan and Boyce (p.227 in 3rd ed. text). Also complete parts (a)–(c), below.

(a) Verify that the two solutions that you have obtained are linearly independent.

(b) Let \(y(1) = y_0, \quad y'(1) = v_0\). Solve the initial value problem. What is the longest interval on which the initial value problem is certain to have a unique twice differentiable solution?

(c) Now suppose that \(y(0) = y_0, \quad y'(0) = v_0\). Is there a solution in this case, and, if so, how does it depend on \(y_0\) and \(v_0\)? If there is a solution, how many are there? Explain.
