Instructions: Solve each of these problems. Your solution should be complete and written out in complete sentences. Where graphs are needed, you may include a print-out of output from Matlab (or another program, if you prefer).

1. Problem 6 in §6.3 of Brannan and Boyce (p.409 in the 3rd ed. text). Also complete parts (a) and (b), below.
   (a) Find a nonzero initial condition for which the solution to the system remains bounded. Explain why solution remains bounded, and sketch the trajectory corresponding to that solution in phase space.
   (b) Find all initial conditions for which solutions to the system remain bounded. These initial conditions live in a plane; what plane is it? Sketch a phase portrait of solution in this plane.

2. Problem 6 in §6.4 of Brannan and Boyce (p.419 in the 3rd ed. text). Also complete part (a) below. (Hint: you may want to use a program like Matlab to find the eigenvalues of the system.)
   (a) Give a formula for purely oscillatory solutions to the system, and the initial conditions that will give those. Sketch the trajectory that starts at \((x(0), y(0), z(0)) = (0, 3, 3)\) in phase space.

3. Problems 14 and 16 in §6.6 of Brannan and Boyce (pp.437–38 in the 3rd ed. text). (Hint: Matlab solves the matrix system \(Ax = b\) for \(x\) with the command \(A \backslash b\).)

4. In lab 5 we consider the Lorenz system,
   \[
   x' = \sigma(-x + y) \quad y' = rx - y - xz \quad z' = -bz + xy.
   \]
   In the following, as in lab, we take \(\sigma = 10\) and \(b = \frac{8}{3}\).
   (a) Find all of the critical points for the Lorenz system in terms of the parameter \(r\).
   (b) Find the Jacobian for the Lorenz system.
   (c) Now, consider \(r = 7\). What are the critical points of the system? Evaluate the Jacobian you found in (b) at the positive nonzero critical point and find its eigenvalues and eigenvectors. (Hint: you will likely want to do this using Matlab or an equivalent program)
   (d) Use your work in (c) to write the general solution to the linearization of the Lorenz equations near the positive critical point. Explain what information this gives us about the original system.
   (e) Use your work from lab to plot the numerical solution trajectory corresponding to the case \(r = 7\) with initial condition \(x(0) = y(0) = z(0) = 0.1\). Plot the solution to the linear system with the same initial conditions to confirm your conclusion in (d).