Math 216 Differential Equations

Lab 1: Graphical Solutions to First Order Differential Equations

Goals

In this lab you will study differential equations and their initial-value problems with the help of `dfield7`, a useful tool for investigating direction fields and solution curves of differential equations. With `dfield7` or similar programs, it is easy to get *qualitative* information about the solutions of a differential equation. Moreover, with a little practice it is also possible to use `dfield7` or similar programs to get *quantitative* information about the solution of initial-value problems for differential equations.

Using Matlab with `dfield7`

After logging on to the Macintosh system in the Instructional Computer Labs located in the basement of East Hall, launch Matlab by clicking on the corresponding icon in the dock. As Matlab starts up, a logo and 3D graph resembling the icon will show up and eventually a window will open with the Matlab prompt. (It is >>). In Windows or other environments the procedure for starting Matlab may be different.

To begin this lab, type `dfield7` in the Command Window and hit return. A window with the title DFIELD7 Setup opens with lots of little boxes all filled in; ignore them for now and click on the Proceed button. You will then see a graph with a direction field entitled $x' = x^2 - t$. Now put the cursor on the point (2,1) and click. Now you know what the solution to the differential equation $x' = x^2 - t$ with initial condition $x(2) = 1$ looks like. Click somewhere else on the graph (that is, try another initial condition). Fool around a bit. Convince yourself that *solution curves do not cross*. (Why not?) If the picture gets crowded you can erase the picture by going to the Edit menu on the DFIELD7 Display window.

Lab problems

1. Let $y(t)$ be the solution curve to the differential equation $y' = \sin(y + t)$ with initial condition $y(0) = 0$. What is $y(5)$? A rough estimate is good enough.

   Answer: $y(5) \approx$ ____

   Note: to do this problem you will have to click on the DFIELD7 Setup window and change the differential equation.

2. Let $y(t)$ be the solution to the differential equation $y' = \sin(y+t)$ with initial condition $y(20) = 0$. What is $y(25)$? A rough estimate is good enough.

   Answer: $y(25) \approx$

   Note: to do this problem you will have to click on the DFIELD7 Setup window and change the minimum and maximum value of t.

3. Let $y(x)$ be the solution to the differential equation $y' = \sin(xy)$ with initial condition $y(0) = 0.1$. What is $y(2)$? A rough estimate is good enough.

   Answer: $y(2) \approx$

   Note: to do this problem you will have to click on the DFIELD7 Setup window and change the equation and also change the name of the independent variable. If you got an error message it may be because xy is written as `x*y` in Matlab; re-type the equation and try again.

4. Let $x(t)$ be the solution curve to the differential equation $x' = (x^2 - 1)\sin(xt)$. Note that in Matlab, x^2 is typed `x^2` so you would enter `x'=(x^2-1)*sin(x*t)`.

   (a) If the initial condition is $x(0) = 1.01$ find both $x(2)$ and $x(-2)$.

   Answer: $x(2) \approx$ and $x(-2) \approx$

   (b) Repeat part (a) but with $x(0) = 1.001$.

   Answer: $x(2) \approx$ and $x(-2) \approx$

   (c) Repeat part (a) but with $x(0) = 1.00001$.

   Answer: $x(2) \approx$ and $x(-2) \approx$

   (d) Repeat part (a) but with $x(0) = 1$.

   Answer: $x(2) \approx$ and $x(-2) \approx$

   (e) Repeat part (a) but with $x(0) = 0.99$.

   Answer: $x(2) \approx$ and $x(-2) \approx$

   How do you solve these problems? You really cannot position the cursor on the graph to 5 decimal place accuracy even if you are steady-handed. However, if you go to the Options menu on the DFIELD7 Display window and choose Keyboard input you can enter the initial condition accurately. Of course, you must also choose appropriate ranges for your variables in the DFIELD7 Setup window. In some problems it doesn't make too

much difference in the final answer if your hand shakes a bit. In others it does. Can you look at the direction field and tell—in advance—which questions you can answer confidently and which are more sensitive?

5. Let $y(x)$ be the solution to the differential equation $y' = y^2 \cos(xy)$ with initial condition $y(0) = 1.217$. To 3 decimal place accuracy what is $y(1)$?

   Answer: $y(1) \approx$

   Note: to do this problem you will have to click on the Edit menu on the DFIELD7 Display window and choose Zoom in; then use the mouse with the button depressed to draw a rectangle you want to enlarge. You can repeat the procedure on the new graph if you need to.

If you have time, you can experiment with some of the other buttons and settings. For example, you can write some text on the graph or plot the direction field with arrows or remove the lines. You can also change the numerical method used; we will study different numerical methods in class, and in Labs 2 and 3.

This concludes Lab 1. If you need to solve a first order differential equation quickly you can do it with the help of Matlab using `dfield7`. What about higher-order differential equations? Systems? Stay tuned!

Note: for this lab, your lab report should simply consist of your own solutions to the problems above and a brief, original, conclusions paragraph.