

Name: _____

SECOND PRACTICE TEST FOR MIDTERM 1, MATH 216

Problem 1.

Answer the following questions.

a) 5 pts. Consider

$$(\tan x)y' + \frac{1}{x+2}y = x^3$$

Fine the largest interval of x containing $x = -1$ where a unique solution of this initial-value problem is guaranteed to exist based on what we have learned in the course. Explain your answer. Do not solve the differential equation.

b) 5 pts Consider the autonomous system $dx/dt = 7x(5-x)$ Draw a phase diagram and determine whether $x = 0$ is a stable or unstable equilibrium.

c) 5 pts Consider the initial value problem

$$dy/dx = -1/(1+x^2), y(0) = 1$$

Suppose we apply the improved Euler method to approximate the solution on the interval $[0, 2]$ with step size $h = 0.02$. Comparing the result of this numerical calculation with the exact solution at $x = 2$ suppose that the cumulative error at $x = 2$ is 0.2. From this information, predict a step size which might be reasonably expected to result in a cumulative error 1/4 that obtained for $h = 0.02$.

d) 5 pts Consider $dy/dx = y^3/x + \sqrt{x^2 + y^2}$, $y(1) = \pi/4$

Let $f(x, y) = y^3/x + \sqrt{x^2 + y^2}$ A student observed that $f(x, y)$ and $f_y(x, y)$ are both continuous in an infinite rectangle R

$$R = \{(x, y); 0 < x, -\infty < y < \infty\}$$

From this, the student concluded that a unique solution to this initial-value problem is guaranteed to exist on the interval $(0, \infty)$ i.e. $0 < x < \infty$. Do you agree or disagree with the student? Circle one and explain your answer.

Problem 2.

10 pts Solve the following initial-value problem

$$xy' - y + 2x^3 = 0, y(1) = 3$$

Problem 3.

10 pts Solve the initial-value problem

$$x' = tx^2 + 2x^2t^2, x(2) = 0,$$

Problem 4.

10 pts Give the explicit solution of the given initial-value problem

$$dy/dt = y(y-6), y(0) = 3.$$

Problem 5.

Consider the differential equation $dx/dt = x^2 + kx$.

- a) 6 pts Find all equilibrium solutions and draw phase diagrams. (Hint: Your answers might depend on k .)
- b) 6 pts Draw a bifurcation diagram and illustrate stable equilibria with solid curves and unstable with dashed curves. Clearly mark any intercepts with axes in your plot.

Problem 6.

Suppose that the engine of a motorboat experienced a problem and stopped working when the motorboat was moving along a lake at a velocity of 35m/sec. Assume that the motorboat encounters air resistance due to the wind of the form $dv/dt = -v^{3/2}$. Let $x(t)$ be a distance measured from the location where the engine stopped.

- a) 5 pts Find the explicit formula for the velocity.
- b) 5 pts Find the distance formula.
- c) 3 pts How far does the motorboat go?

Problem 7.

It is known that the population T of a certain type of tumor cells satisfies the following differential equation

$$dT/dt = 40T^2 - 1600T$$

- a) 8 pts Sketch the solution curves for this differential equation in the (t, T) plane.
- b) 4 pts Without solving the equation, explain what would happen if there were initially 30 thousand tumor cells in the body.