

Name: \_\_\_\_\_

Section/Time of lecture: \_\_\_\_\_

Professor/GSI: \_\_\_\_\_

**December 13, 8-10 am, Aud C, Angell Hall**

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Each part of a problem counts equally. To get full score you need to carefully explain what you did.

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Problem	Points	Score
1	8	
2	8	
3	8	
4	8	
+	3	3
TOTAL	35	

**Problem 1.**

a)

Verify that  $y = e^x - e^{-x}$  satisfies the differential equation  $y' = y + 2e^{-x}$ .

b) Solve

Solve  $\frac{dy}{dx} = x\sqrt{x^2 + 9}$ ,  $y(-4) = 0$ .

c) Solve  $\frac{dy}{dx} = (8xy)^{\frac{1}{3}}$ .

d) Solve  $(x^2 + 4)y' + 3xy = x$ ,  $y(0) = 2$ .

**Problem 2.**

- a) Solve  $y'' + 8y' + 25y = 0$ .
- b) Find a particular solution of  $2y'' + 4y' + 7y = 3x^2$ .
- c) Find the matrix products  $AB$  and  $BA$  if they are meaningful.
- d) Solve

$$\begin{aligned}x' &= 7x - 5y \\y' &= 4x + 3y\end{aligned}$$

**Problem 3.**

a) Decide on stability for the critical point  $(0, 0)$ :

$$\begin{aligned}x' &= y \\y' &= -x\end{aligned}$$

b) Find all critical points and decide stability:

$$\begin{aligned}x' &= x + x^2 + xy \\y' &= y - y^2 - xy\end{aligned}$$

c) Calculate the Laplace transform of the function:

d) Solve

$$\begin{aligned}x'' + 2x + 4y &= 0 \\y'' + x + 2y &= 0 \\x(0) &= 0 \\y(0) &= 0 \\x'(0) &= -1 \\y'(0) &= -1\end{aligned}$$

**Problem 4.** Include table

a) Recall that  $L(e^{at}f(t)) = F(s - a)$  and  $L(t^n) = n!/s^{n+1}$ . Find  $Lf$  if  $f = e^{5t}t^4$ .

b) Let  $f(t) = t$  and  $g(t) = t - 5$ . Find  $f * g$ .

c) Recall that  $L(u(t - a)f(t - a)) = e^{-as}F(s)$ . Find  $f$  if  $L(f) = \frac{e^{-3s}}{s^2}$ .

d) Solve

$$x'' + 4x' + 5x = \delta(t - \pi) + \delta(t - 2\pi)$$

$$x(0) = 0$$

$$x'(0) = 1.$$