Name: _____

Section/Time of lecture: _____

Professor/GSI: _____

December 15, 8-10 am, Aud D, Angell Hall

Each part of a problem counts equally. To get full score you need to carefully explain what you did.

LAPLACE TRANSFORM TABLE IS IN THE BACK

Problem	Points	Score
1	6	
2	9	
3	9	
4	9	
+	2	2
TOTAL	35	

Problem 1.

a) Solve $\frac{dy}{dx} = 2e^{2x-y}, y(0) = 1.$

b) Solve $y'' - 6y' + 25y = 0, y(0) = 1, y'(0) = 2$

c) Solve $y'' + y = \sin x, y(0) = 1, y'(0) = -1.$

Problem 2.

a) Find the general solution of

$$\begin{aligned} x' &= 2x - 3y + 2\sin t \\ y' &= x - 2y - \cos t \end{aligned}$$

using the method of elimination.

b) Use the eigenvalue method to solve:

$$\begin{aligned} x' &= 4x - 3y, x(0) = 1 \\ y' &= 3x + 4y, y(0) = -1 \end{aligned}$$

c) Decide on stability at the origin of

$$\begin{aligned} x' &= 2x - y \\ y' &= x - 3y \end{aligned}$$

Problem 3.

a) Find all the critical points and decide stability at each:

$$\begin{aligned} x' &= 60x - 3x^2 - 4xy \\ y' &= 42y - 3y^2 - 2xy \end{aligned}$$

b) If $f(t) = e^{3t}$, then the Laplace transform $F(s) = \frac{1}{s-3}$. Verify this formula directly from the definition of Laplace transform $L(f(t)) = \int_0^\infty e^{-st} f(t)dt$.

c)

$$x'' + 3x' + 4x = t^2 + \cos(3-2), x(0) = 1, x'(0) = 2.$$

Write down a formula for the Laplace transform of $x(t)$. Don't solve for $x(t)$.

Problem 4.

a) Let $f(t) = e^{3t}, g(t) = e^{2t}$. Calculate the convolution $f * g$.

b) Suppose that the Laplace transform of a function $g(t)$ is

$$F(s) = \frac{s(1 - e^{-2s})}{s^2 + \pi^2}.$$

Find $g(t)$.

c) Solve the equation

$$x'' + 2x' + 2x = 2\delta(t - \pi).$$