1. In lab 3 we consider the nonlinear system

\[ N' = \gamma(A - N(1 + P)), \quad P' = P(N - 1). \]

We continue our analysis of this system here.

(a) Find all critical points of this system.

(b) Find a linear system that approximates the nonlinear system at each of the critical points you found in (a). To do this, let \((N_0, P_0) + (u, v)\) (where \((N_0, P_0)\) is a critical point, and \(|u, v| \ll 1\). Plug into the equation and discard nonlinear terms (which may be considered to be negligible).

(c) Determine the type and stability of the critical point \((1, A - 1)\) in the cases \(0 < A < 1\) and \(A > 1\). You may assume that \(\gamma < \frac{4(A - 1)}{A^2}\) when \(A > 1\).

(d) Is there a value of \(\gamma\) for which the linear system has a repeated eigenvalue? Sketch a phase portrait in this case.


3. Problem 24 in §4.2 of Brannan and Boyce (p.227 in the 3rd ed. text). Also complete parts (a) and (b), below.

(a) Let \(y(1) = y_0, \ y'(1) = v_0\). What is the longest interval on which the initial value problem is certain to have a unique twice differentiable solution?

(b) Now let \(y(0) = y_0, \ y'(0) = v_0\). Can you find a solution to this initial value problem? A unique solution? Explain.

4. Problem 48 in §4.3 of Brannan and Boyce (p.240 in the 3rd ed. text). Also complete parts (a) and (b), below.

(a) For what value of \(\alpha\), if any, could the phase portrait for the equation have a single straight line solution?

(b) For the case you found in (a), show that it is not possible for a solution to cross the \(t\) axis more than once.