1. Problem 36 in §4.5 of Brannan and Boyce (p.261 in 3rd ed. text). Also complete parts (a) and (b), below.
   (a) Suppose that \( \lambda = N\pi \). Find the general solution in this case. Explain how it is different from your solution of the original problem.
   (b) Suppose that the operator is \( L[y] = y'' + y' + y \), instead of \( L[y] = y'' + \lambda^2 y \). Find the particular solution in this case.

2. Problem 24 in §4.6 of Brannan and Boyce (p.273 in 3rd ed. text). For part (c), find the solution to the linear spring problem (which will be \( y'' + 0.2y' + y = \cos(\omega t) \)) analytically—that is, by hand when finding the gain function \( G(\omega) \). You should graph the solution curves from a numerical solution instead.

3. Problem 16 in §5.1 of Brannan and Boyce (p.303 in 3rd ed. text). Use the integral definition of the Laplace transform to find \( L\{f(t)\} \). Also complete parts (a)-(c), below.
   (a) Suppose we are solving \( y'' + 2y' + y = f(t) \) for this \( f(t) \), with the initial conditions \( y(0) = 0, \ y'(0) = 1 \). Solve this without using Laplace transforms.
   (b) Find an expression for \( Y(s) = L\{y(t)\} \) for the problem \( y'' + 2y' + y = f(t), \ y(0) = 0, \ y'(0) = 1 \), using your work from problem 16.
   (c) Compare the solution you found in part (a) with the transform that you found in part (b). Identify which parts of the solution match the different terms in the Laplace transform.

4. In lab 4 we consider the differential equation \( y'' + 2\gamma y' + \omega^2_0 y = F(t) \) for different forcing terms \( F(t) \). In this problem we analyze this equation further using Laplace transforms.
   (a) Consider \( y'' + y' + 40y = I(t) \), where \( I(t) = \begin{cases} 0, & t < 1 \\ 1, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases} \). Find the forward transform \( Y = L\{y\} \) if \( y(0) = y'(0) = 0 \).
   (b) Solve \( y'' + y' + 40y = 1, \ y(0) = y'(0) = 0 \), using Laplace transforms. Notice how the value of \( Y(s) \) you obtain differs from that you found in (a). Plot your solution for \( 0 \leq t \leq 5 \).
(c) Now suppose that in the problem you solve in part (b) we turn off the forcing term at $t = 2$ (that is, we set it to zero). Sketch, by hand, the resulting solution you would expect for $t \geq 2$ (you may want to just add this to the plot you obtained in (b)). Explain why your solution has the form it does.

(d) Solve $y'' + y' + 40y = 1$, $y(0) = 2$, $y'(0) = 3$, using Laplace transforms. Plot your solution.