

Name: \_\_\_\_\_

Section/Time of lecture: \_\_\_\_\_

Professor/GSI: \_\_\_\_\_

## MIDTERM II

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Each part of a problem counts equally. To get full score you need to carefully explain what you did.

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Problem	Points	Score
1	6	
2	6	
3	6	
4	7	
TOTAL	25	

**Problem 1.**

a) Find a particular solution of

$$y'' + 6y' + 9y = x^3$$

b) Find the general solution of

$$y'' + 6y' + 9y = e^{-3x}$$

c)

Use the method of variation of parameters to find a particular solution of

$$y'' - 4y' + 4y = e^{2x}$$

**Problem 2.**

a)

Write the equation

$$x'' - 11x' + 2x = t \ln t$$

as a system of first order equations.

b) Find the general solution of the system

$$\begin{aligned}x' &= 2y \\y' &= -7x\end{aligned}$$

using the method of elimination.

c) Find the solution of

$$\begin{aligned}x' &= 2x - y, x(0) = 1 \\y' &= 5x + y, y(0) = -1\end{aligned}$$

using the method of elimination.

**Problem 3.** a)  
The matrices

$$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$$

Calculate  $AB$  and find  $\det(AB)$ .  
b) Find the eigenvalues of

$$\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$$

c) Find the eigenvectors of

$$\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$$

**Problem 4.**

a) Find the general solution of the system

$$\begin{aligned}x_1' &= 2x_1 + 3x_2 \\x_2' &= 2x_1 + x_2\end{aligned}$$

using the eigenvalue method.

b) Find eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$$

c) Find the general solution of the system

$$\begin{aligned}x_1' &= 3x_1 - 4x_2 \\x_2' &= 4x_1 + 3x_2\end{aligned}$$