IRS

Consider the (solitaire) game of “IRS”, which is played as follows:

- A complete set of integers from 1 to $N$ is given. This is the board. (For example, take $N = 100$)

- A legal move is to remove some integer $m$ remaining on the board, as long as there is some proper divisor $m_1$ of $m$ also remaining on the board. When this is done, you get the integer $m$ and the IRS gets all the integers remaining on the board that strictly divide $m$. (Thus you get one integer and the IRS gets at least one integer after every move, possibly more than one integer.)

- When no more legal moves are possible, the IRS gets all the remaining integers on the board.

- Your score at the end of the game is the sum of the integers you took. The IRS’s score is the sum of all the integers the IRS got. You win if you score higher than the IRS.

For example on the integers up to $N = 10$ a game could go: You choose 7, the IRS gets 1, you choose 8, the IRS gets 4 and 2; you choose 10, the IRS gets 5; you choose 6 the IRS gets 3; no legal moves remain, the IRS gets 9. Your total $6 + 7 + 8 + 10 = 31$, the IRS gets $1 + 2 + 3 + 4 + 5 + 9 = 24$, so you win. Pretty easy, huh?

Some questions:

1. Find approximate (“good”) strategies for playing the game.

2. Implement various strategies on a computer.

3. Who wins the game with best play? (This has been determined for small $N$ but it is not solved in general. How far can you resolve it?)

Consider one or more generalizations of the problem, such as:

- Suppose there are 2 players against the IRS, that alternate turns. Can they cooperate so that one or both of them beat the IRS? Or does the IRS always win?

- Propose your own generalizations. Vary the rules, number of players, the initial set of numbers on the board (allow nonconsecutive integers) etc.