Example 2: Mathematics

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There are many ways to write formulas in \LaTeX. Inline formulas are delimited by dollar signs. For example, \( z^2 = \sin(\theta_0) \), or \( 2 \times 2 \neq 5 \).

Here are three ways to display equations without a reference number:

\[
\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, dt
\]

\[
1 + (-1)^n = \begin{cases} 2 & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases}
\]

An equation with reference number:

\[
\lim_{n \to \infty} \sum_{k=1}^n \frac{1}{k^2} = \frac{\pi^2}{6}.
\tag{1}
\]

\LaTeX{} automatically adds the number. The equation can later be referenced: See equation (1). We can also get the page the equation appears on, in this case it is page 1.

Long lines can be split:

\[
\langle (W_j^{\text{asr}})^2 \rangle_q = \frac{1}{(\log q)^2} \left( 2 \sum_{n_1=2}^{q-1} \frac{\Lambda(n_1)^2}{n_1} \frac{1}{\log q} \right)^2
\]

\[
+ 2 \sum_{n_1=2}^{q-1} \frac{\Lambda(n_1) \Lambda(\bar{n}_1)}{\sqrt{n_1} \sqrt{\bar{n}_1}} \frac{1}{\log q} \frac{1}{\log q} \left( \frac{\log n_1}{\log q} \right) \left( \frac{\log \bar{n}_1}{\log q} \right)
\]

\[
+ O \left( \frac{1}{q-2} \left( \frac{1}{\log q} \sum_{n \leq q} \frac{\Lambda(n)}{\sqrt{n}} \right)^2 \right)
\tag{2}
\]

To group several equations, aligning on the = sign, use the \texttt{align} command. The computer aligns at the \&. The \texttt{\nonumber} command stops the equation number appearing on that line.

\[
y = (x + 1)^2 - 1
\]

\[
= x^2 + 2x + 1 - 1
\]

\[
= x(x + 2).
\tag{3}
\]
Macros are useful to

1. Cut down on the amount of typing required, for example

\[ J_{k-1} \left( 4\pi \sqrt{\frac{m^2 p_1 p_2 N}{c}} \right) \]

2. Keep consistency in definitions. For example, if \( E \) denoted the error term in the second moment of the Riemann zeta function,

\[
E(T) := \int_0^T \left| \zeta\left(\frac{1}{2} + it\right) \right|^2 \, dt - T \left( \log \frac{T}{2\pi} + 2\gamma - 1 \right)
\]

and it was later decided this should instead be \( E_2(T) \), changing the macro \texttt{newcommand(\texttt{\{E_2\})}} to change this throughout the entire document.

One should define new function names using \texttt{DeclareMathOperator}, as this will give the correct font and spacing. Compare $\text{li}(x)$ which yields \textit{li}(x) and $\text{	extbackslash li}(x)$ which yields li(x). For example

\[ \text{Res}_{s=1} \zeta(s) = 1 \]

Examples of a theorem, corollary and conjecture are given below. As before they can be referenced: See Theorem 1.

**Theorem 1** (Prime Number Theorem). Let \( \pi(x) \) denote the number of primes up to \( x \). As \( x \to \infty \)

\[ \pi(x) \sim \frac{x}{\log x}. \]  \tag{4}

Let

\[ \text{li}(x) = \int_2^x \frac{1}{\log t} \, dt \] \tag{5}

**Corollary 1.1.** \( \pi(x) \sim \text{li}(x) \).

**Proof.** Integrating by parts, we see that \( \text{li}(x) = \frac{x}{\log x} + O\left(\frac{x}{(\log x)^2}\right) \), so Theorem 1 implies \( \pi(x) \sim \text{li}(x) \), as required. \qed

**Conjecture 2.**

\[ \pi(x) = \text{li}(x) + O\left(x^{1/2} \log x\right) \] \tag{6}

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