Some Facts About Integrals

Integration is an integral (pun intended) part of calculus. Here are just a few of the interesting facts about integrals.

- There are multiple kinds of integration
  - Riemann integration
  - Darboux integration
  - Lebesgue integration
  - Many others.
- Integrals can be used to tell you the “area under a curve.”
- Integrals are linear.

For now, we will focus on this last point, that integrals are linear.

1.1 Linearity of Integrals

What does it mean to say that integrals are linear? Well, if \( f_1, \ldots, f_n \) are integrable functions defined on \((a, b)\) where \(a\) and \(b\) are real numbers, then

\[
\sum_{i=1}^{n} \int_{a}^{b} f_i(x) \, dx = \int_{a}^{b} \sum_{i=1}^{n} f_i(x) \, dx.
\]

2 An important theorem

One of the most important theorems involving integrals is the fundamental theorem of calculus, the second part of which is presented below.

**Theorem 1.** If \( f \) and \( g \) are real valued functions such that for all \( x \) in \([a, b]\), \( f(x) = g'(x) \), then

\[
\int_{a}^{b} f(x) \, dx = g(b) - g(a).
\]

**Proof.** See any decent calculus book.
3 Examples

Here are some example problems for you to try. (You may find the result of Theorem 1 useful.)

1. $\int_{1}^{4} (\sqrt{2}x + \cos(x^2 + 1)) \, dx =$

2. $\int_{-5}^{15} \frac{1}{6} \, dx + \int_{-5}^{15} (14x + 2) \, dx =$

To help you out, here is a table of some common derivatives:

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$df/dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin(x)$</td>
<td>$\cos(x)$</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>$x^n$</td>
<td>$nx^{n-1}$</td>
</tr>
</tbody>
</table>