1) a) For which, if any, elements of a group $G$ is multiplication by that element a group homomorphism, i.e. for which $a \in G$ is the map $G \to G$ given by $g \mapsto ag$ a group homomorphism? 
b) For which, if any, groups $G$ is inversion $(g \mapsto g^{-1})$ a group homomorphism? 
c) For which, if any, groups $G$ is “squaring” $(g \mapsto g^2)$ a group homomorphism? 

2) For each case below, list the distinct left and right cosets of $H$ in $G$. Is $H$ normal in $G$?

   a) The subgroup $H = \langle [3] \rangle$ of $G = \mathbb{Z}_{12}$.
   b) The subgroup $H$ generated by $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ in $G = S_3$.

3) What are the possible orders of subgroups of $S_4$? For each possibility, find a subgroup of $S_4$ having that order.
   Hint: When constructing larger subgroups, it may be helpful to think about the symmetries of a tetrahedron (the “pyramid-like” shape built from 4 equilateral triangles).

4) Suppose that $p$ and $q$ are positive primes and that $G$ is a finite group of order $pq$. Show that every proper subgroup of $G$ is cyclic.

5) If $f : G \to H$ is a group homomorphism and $N$ is a normal subgroup of $G$, prove that $f(N)$ is a normal subgroup of $f(G)$.

6) Give an example of groups $G$, $K$, and $H$ such that $H$ is a normal subgroup of $K$ and $K$ is a normal subgroup of $G$ but $H$ is not a normal subgroup of $G$. (Hint: Consider subgroups of $D_4$.)

7) Let $G$ be a group and let $Z = Z(G)$ be the center of $G$.
   a) Prove that $Z$ is normal in $G$.
   b) Prove that $G/Z$ is cyclic if and only if $G$ is abelian.
   c) Give an example of a nonabelian group $G$ such that $G/Z$ is abelian.

8) Let $G$ be the group consisting of all matrices of the form
   \[
   \begin{pmatrix}
   1 & a & b \\
   0 & 1 & c \\
   0 & 0 & 1
   \end{pmatrix}
   \]
   $(a, b, c \in \mathbb{Q})$
   with multiplication as the group operation.
a) Find $Z(G)$ (the center of $G$). Show that it is isomorphic to $\mathbb{Q}$.

b) Show that $G/Z(G)$ is isomorphic to the group $\mathbb{Q} \times \mathbb{Q}$. 