1) Prove or disprove: if $T$ and $S$ are subrings of a ring $R$ then so is $T \cap S$.

2) Prove the following statements.
   i) Let $a$ and $b$ be non-zero integers. Show that $(a, b + ta) = (a, b)$ for any integer $t$.
   ii) If $r | ab$ and $(a, r) = 1$ then $r | b$. 
3) Find the inverse of \([17]\) in \(\mathbb{Z}_{25}\).

4) Consider the set \(\mathbb{R} \times \mathbb{R}\). We make this set into a ring using the following operations:

\[(a, b) + (c, d) = (a + c, b + d)\]
\[(a, b) \cdot (c, d) = (ac - bd, ad + bc)\]

Show that with these operations \(\mathbb{R} \times \mathbb{R}\) is a field.
5) Let $R$ be a ring. Show (directly from the axioms) that $0 \cdot r = 0$ and $r \cdot 0 = 0$ for any element $r \in R$.

6) Give an example of:
   – a commutative ring without an identity, and
   – a commutative ring with identity that is not an integral domain.