Lesson 24: Radians

(Cover 8.1 and (parts of) 8.3) Read: Section 8.2
Announce: EXAM 2 is Tuesday, March 20 at 6 pm in East Hall 1324

Exam 2 will focus on Sections 4.3, 4.5, 5.1-5.3, 6.1-6.5, 7.1-7.4, 8.1, 8.2, and 8.3 (pages 334-336).

The most important points and skills for §8.1

- Given an angle expressed in radians and a unit circle, students are able to locate the point on the unit circle that corresponds to the given angle. For example, students are able to do this mentally moving around the circle in a counter-clockwise (clockwise if the angle is negative) direction from the point (1, 0) until they have travelled a distance on the rim of the unit circle that is (approximately) equal to the number of radians.

- Given an angle (positive or negative) expressed in radians and a unit circle, students are able to evaluate or approximate the cosine (resp. sine) of the angle. For example, students are able to do this by locating the point on the rim of the unit circle that corresponds to the angle and then measuring or estimating the $x$–coordinate (resp. $y$–coordinate) of this point.

- Given an angle described in degrees, students are able to convert the angle to radians (and vice versa).

- Given an angle expressed in radians, students are able to use a calculator to find the sine and the cosine of the angle and interpret these as the coordinates of a point on the unit circle.

- Given information about two of the three quantities (arc length, radius or angle) students are able to use this information together with the arc length formula to calculate the value of the third quantity.

The most important points and skills for §8.3
(Note that we do NOT cover secant, cosecant, or cotangent in this course.)

- Students should recognize the relationships between sine and cosine coming from symmetries and shifts of their graphs.

- Students should know the Pythagorean Identity.

Lesson 24 Supplement: There is a version of the chart of “special values” from Lesson 22 that includes radians. If you so choose, you can use this in class or hand it out as something students can fill out and study.

Comment: Many students are a lot happier when working with degrees rather than with radians for angle measure, although radian measure is what we want the students to get comfortable with in Math 105. Many students will have met radians before but simply prefer to work with degrees rather than with radians. The reason for insisting that they work in radians except where instructed otherwise is mainly because radians will be the default angle measure if/when the students go on to calculus.
Suggested Lesson Plan:

00–20 Use this time for a quiz or to do additional examples that you did not get to last time.

Summarize the last lesson in 1-2 sentences. Outline today’s lesson.

20–35 Move now from degrees to radians. Explain that radians are merely another way of locating the stopping point for a particular rotation, but in this case we describe the point by giving its distance from the point (1, 0) along the rim of the circle, rather than by the angle at the center.

As part of your mini-lecture you could have students convert some angles from degrees to radians (use the angles you have previously located on the unit circle in terms of degrees, and add any missing “common angles” (0°, 30°, 45°, 60°, and 90°). It is probably worth writing up the conversion formulas for students during your mini-lecture, i.e.

\begin{align*}
\text{Degrees to Radians: } \theta_{\text{Radians}} &= \frac{2\pi}{360} \cdot \theta_{\text{Degrees}} \\
\text{Radians to Degrees: } \theta_{\text{Degrees}} &= \frac{360}{2\pi} \cdot \theta_{\text{Radians}}
\end{align*}

and even better if you can explain how these conversion formulas are arrived at so that students may be able to figure them out from “first principles.”

Have the students practice converting a few angles. Choose from Section 8.1 # 3, 6, 9, and 11 on page 322 for a few quick exercises.

During this part of the lesson, you could also ask the students to work a problem like some parts of Section 8.1 Problem #15 on page 322 to get them thinking in radian mode and to be sure they can begin to visualize the stopping point for an angle of 3 radians, etc. This problem helps to stress that the radian definition gives us the distance along the rim of the unit circle.

35-50 Next introduce the relationship between arc length, circle radius, and angle measurement in radians. Have the students work on Section 8.1 #27 and #26 (in that order!!) on page 322 in their groups. When going over the answers, make sure to draw these arcs and circles on the board.

50–65 At this point, remind students that the radian measurement is more commonly used than the degree measurement. They should know that radians are assumed, unless degrees are specifically noted. (Make sure that they know how to set their calculators to RADIANS and DEGREE modes.) In particular stress that the expression cos(3) - without units, and in particular without the degree sign-indicates the cosine of 3 radians.

Recall from the last lesson that the coordinates of point P on a circle of radius r centered at the origin are determined by \( x = r \cos(t) \) and \( y = r \sin(t) \), where the angle \( t \) is measured clockwise from the positive \( x \)-axis. Have the students work on Section 8.1 #32 on page 322 in their groups. Then have them do Section 8.1 #41 on page 262 in their groups. For #41, be sure to strongly encourage the use of a picture!

65–80 Now, sketch the graphs of sine and cosine using radians on the horizontal axis, and remind the students of the key features of (the horizontally stretched versions of) these graphs that
they found last time. Note that the periods of these functions are now $2\pi$. Point out that the two graphs are horizontal shifts of each other. Write down the relationships you can read from this, e.g. $\sin(x + \frac{\pi}{2}) = \cos(x)$ since the graph of $\cos(x)$ can be obtained from that of $\sin(x)$ by shifting left $\pi/2$ units.

Remind students of the Pythagorean Identity $\sin^2 t + \cos^2 t = 1$ and use it for an example like Section 8.3 #28 on page 340.

Summarize the key points of the lesson.

**If you have extra time...** you can move on to some basic transformations of the sine and cosine functions as an introduction to Section 8.2.