UNIVERSITY OF MICHIGAN

DEPARTMENT OF MATHEMATICS

Qualifying Review Examination in Topology

3 January 2007: Morning Session, 9:00 -12:00

1. Let $f : X \to Y$ be a continuous closed surjection such that $f^{-1}(y)$ is compact for all $y \in Y$. Suppose that X is Hausdorff. Prove that Y is Hausdorff.

2. Let X be the set of real 2×2 matrices with determinant = 3 considered as a subspace of R^4. Is X a manifold? Prove that your answer is correct.

3. Let X be the space obtained by attaching a Moebius strip M to a torus $T = S^1 \times S^1$ by a homeomorphism of the boundary circle of M to the circle $S^1 \times \{(1, 0)\}$ in T (where (1, 0) denotes a point of the standard S^1 in R^2). Calculate all the homology groups of X .

4. Let the topological space X_n be obtained from S^n by identifying three distinct points, i.e. $X_n = S^n / \{p, q, r\}$. Find the fundamental group of X_n.

5. Let $q : X \to Y$ be a quotient map of X onto a connected space Y . Assume that $q^{-1}(y)$ is connected for each $y \in Y$.

a. Show that X must be connected.
b. Is X necessarily connected if the map q is only assumed to be continuous and onto and again assuming that $q^{-1}(y)$ is connected for each $y \in Y$? Explain.

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3 January 2007: Afternoon Session, 2:00 -5:00

1(a) Show that a connected locally path connected space is path connected.

(b) Show that there are connected spaces which are not path connected.

2. a. Let $f : S^2 \to R^3$ be a smooth embedding. Prove that there exist distinct points $x, y \in S^2$ such that the tangent planes to $f(S^2)$ at $f(x)$ and $f(y)$ are parallel.

b. Exhibit a smooth proper embedding $g : R^2 \to R^3$ such that if x and y are any two distinct points in R^2 then the tangent planes to $g(R^2)$ at $g(x)$ and $g(y)$ are **not** parallel. (A continuous map is proper if the pre-image of any compact set is compact.)

3. Let A and B be two round circles in R^3 which intersect in a single point. Compute all the homology groups of $R^3 - (A \cup B)$.

4. Show that every continuous map $f : RP^2 \to RP^2$ has a fixed point.

5. Define $f : R^1 \to S^1$ by $f(x) = e^{i(x - \sqrt{2}\sin(x/\sqrt{2}))}$. Find all the regular points, all the regular values, all the critical points and all the critical values of f .