UNIVERSITY OF MICHIGAN

DEPARTMENT OF MATHEMATICS

Qualifying Review Examination in Topology

1 September 2007: Morning Session, 9:00 -12:00

1. Let X be a topological space and let $d : X \times X \to R$ be continuous. Suppose that d satisfies the axioms for a metric, that is,

i) for all x and y in X, d(x,y) ≥ 0 and d(x,y) = 0 iff x = y
ii) for all x and y in X, d(x,y) = d(y,x)
iii) for all x, y and z in X, d(x,y) + d(y,z) ≥ d(x,z).

Prove or give a counterexample to the statements:

a) The topology of X is given by the metric d.

b) X is metrizable.

2. Find a CW complex X such that $H_0(X) = Z,\ H_5(X) = Z \oplus Z_3,\ H_n(X) = 0$, for n ≠ 0 or 5.

3. Let M = {(x,y,z,w) $\in$ R^4 | $x^4 + y^4 + z^2 + w^2 = 1$} and let $f : M \to R$ be given by $f(x, y, z, w) = x^3 - z.$

a) Show that M is a manifold.
b) Find the critical points of f.

4.Let X = $RP^2 \times S^1$. Calculate $\pi_1(X)$ and $H_*(X)$ without using the Kunneth theorem.

5. Let Γ: $R \to R^2$ be a smooth curve and let A be the set of r > 0 such that the circle of radius r about the origin is tangent to Γ at some point. Show that the interior of A is empty, that is, A does not contain any open interval in R.

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Qualifying Review Examination in Topology

1 September 2007: Afternoon Session, 2:00 -5:00

1. Let $R^n \subset R^{n+1}$ by $(x_1,...x_n) \mapsto (x_1,...x_n,0)$ and let $R^\infty = \bigcup_{n=1}^{\infty} R^n$. Similarly, let $\Sigma = \bigcup_{n=1}^{\infty} S^n$. Define a topology on $\Sigma \subset R^\infty$ by $C \subset \Sigma$ is closed iff $C \cap S^n$ is closed in S^n for every n = 1, 2, …

Prove or disprove the following statements:

a) Σ is connected.
b) Σ is compact.
c) Σ is locally compact.

2. Let f : $S^1 \to X$ and g : $S^1 \to X$ be continuous maps and suppose that f and g are homotopic. Suppose also that f(1) = g(1) = x_0 where 1 is the basepoint in S^1 and x_0 is a point in X. Show that [f] is conjugate to [g] in $\pi_1(X,x_0)$.

3. Let X be the set of <u>pairs</u> of <u>unit</u> vectors $(\vec{x},\vec{y})$ in R^3 such that $\vec{x}\bullet\vec{y} = \frac{1}{2}$ where $\bullet$ denotes the dot product. Thus $X = \{(\vec{x},\vec{y}) \in R^3 x R^3 \mid \ |\vec{x}| = 1, \ |\vec{y}| = 1, \ \vec{x}\bullet\vec{y} = \frac{1}{2}\}$.
Prove that X is a manifold.

4. Let K be a simplicial complex that is homeomorphic to S^7 and let i : L $\subset$ K be a subcomplex that is homeomorphic to RP^2. Let X = K $\cup$ L x I/~ where (x, 0) ~ i(x) and (x, 1) ~ (x', 1) for all x and x' in L. Calculate $H_*(X)$.

5.Let $X = R^\omega = \prod_{n=1}^{\infty} R$ with the product topology and let $Y = R^\omega = \prod_{n=1}^{\infty} R$ with the box topology. Prove that X and Y are not homeomorphic.