UNIVERSITY OF MICHIGAN

DEPARTMENT OF MATHEMATICS

Qualifying Review Examination in Topology

6 January 2008: Morning Session, 9:00 -12:00

1) Let $D^3 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 \le 1\}$, let $Y = \{(x,0,\frac{1}{2}) \in \mathbb{R}^3 \mid x \in \mathbb{R}\} \cup \{(0,0,z) \in \mathbb{R}^3 \mid z < \frac{1}{2}\}$ and let $X = D^3 \cap (\mathbb{R}^3 - Y)$. That is, X is a 3-ball with a "T" removed.

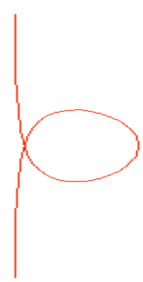
Compute a) $\pi_1(X,*)$ and b) $H_*(X, Z)$

2) Let $f : X \to Y$ be continuous. Show that for every set $Z \subset X$, $f(\overline{Z}) \subset \overline{f(Z)}$.

3) Let $p : \widetilde{X} \to X$ be an n-sheet covering space of a finite CW complex. Prove that $\chi(\widetilde{X}) = n\chi(X)$. Use this result to determine, up to isomorphism, ***all*** two sheeted covering spaces of the Klein bottle. You may use the theorem on classification of closed surfaces.

4) Consider the space $V_{n+1,2} = \{(x_1,x_2) \in S^n \times S^n \mid \langle x_1,x_2 \rangle = 0\}$ where S^n is the unit sphere in the Euclidean space $\mathbb{R}^{n+1}$ with the standard inner product. Denote by $p : V_{n+1,2} \to S^n$ the projection on the first factor; $p(x_1,x_2) = x_1$. Prove that there is a section $s : S^n \to V_{n+1,2}$ if and only if n is odd.
A section of the map p is a continuous function s such that $p(s(x_1)) = x_1$.

5) We consider **immersions** f of R into R^2 with the property that f(t) = (0,t) for |t| > 1.
Examples: a) $f_0(t) = (0, t)$ b) f_1 given by sketch below.



Let F' (t, s) denote the vector $\dfrac{\partial F(t,s)}{\partial t}$. Show that there does not exist a homotopy $F : R \times I \to R^2$ such that F' is continuous, F(t, s) = (0, t) for |t| > 1, $F(t, 0) = f_0(t)$, $F(t,1) = f_1(t)$ and $F'(t, s) \neq 0$ for all (t, s) in R x I.

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Qualifying Review Examination in Topology

6 January 2008: Afternoon Session, 2:00 -5:00

1) Let $T^2 \subset S^3$ be the standard embedded torus in S^3. Let K $\subset T^2$ be the (2, 3) torus knot given by the image of $\gamma : \mathbb{R} \to T^2 = S^1 \times S^1, \gamma(t) = (e^{i2t}, e^{i3t})$.
Compute a) $\pi_1(S^3 - K)$ and b) $H_*(S^3 - K, Z)$.

2) Let M be a compact smooth manifold of dimension n and let $f : M \to \mathbb{R}^n$ be a smooth map.
Show that f cannot be everywhere nonsingular.

3) Describe the homotopy classes of maps from $(\mathrm{RP}^2, *)$ to $(\mathrm{S}^1 \times \mathrm{S}^3, *')$ where $*$ and $*'$ are base points.
Explain your answer clearly.

4) Suppose that X and Y, submanifolds of R^3, are transverse regular and that X is diffeomorphic to S^1 and Y is diffeomorphic to $\mathrm{S}^1 \times \mathrm{S}^1$.
Show that $\mathrm{X} \cap \mathrm{Y}$ is a finite set with an even number of elements.

5) A function f : X $\to$ Y is said to be locally constant if, for each x in X, there is a neighborhood U(x) of x such that f(y) = f(x) for each y in U(x). Choose the ***one*** assumption below that will enable you to prove, with that assumption, that a locally constant function is constant, then prove it.

a) X is compact b) X is connected c) X is second countable
d) X is locally connected e) Y is normal.