

UNIVERSITY OF MICHIGAN
DEPARTMENT OF MATHEMATICS
Qualifying Review Examination in Topology
1 September 2008: Morning Session, 9:00-12:00

- (1) Let X be $\prod_{i=1}^{\infty} \mathbf{R}_i$, where each \mathbf{R}_i is the Euclidean real line. Generate the topology on X from basis sets of the form $\prod_{i=1}^{\infty} U_i$, where each U_i is open in \mathbf{R}_i .
- Is X Hausdorff?
 - Is X connected?
 - Is X locally compact?
 - Does X have a countable dense subset?
- Justify your answer.
- (2) Let $X = S^1 \vee S^1$, the wedge of two circles, i.e., "figure eight". If $\phi : X \rightarrow S^3$ is an embedding of X into a 2-sphere in the 3-sphere S^3 , compute $H_*(S^3 - \phi(X), \mathbf{Z})$.
- (3) If we express S^3 as the union of two connected nonempty open subsets X, Y , then $X \cap Y$ is always connected.
- (4) Assume that M and N are smooth manifolds without boundary, that M is compact, and that N is non-compact and connected. Show that every smooth map $f : M \rightarrow N$ has at least one critical point.
- (5) Let S_g denote the oriented surface of genus $g \geq 0$. Let $f : S_g \rightarrow S_g$ be a map which is homotopic to the identity. For each g , does f necessarily have a fixed point? Your answer can depend on g .

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1 September 2008: Afternoon Session, 2:00-5:00

- (6) Let X and Y be Hausdorff topological spaces such that Y is compact. Let $p : X \times Y \rightarrow X$ be the projection onto the first factor. Show that p maps each closed subset of $X \times Y$ to a closed subset of X .
- (7) Using covering space technique, find all the subgroups of index two of F_2 , the free group of rank two.
- (8) Let $i : S^1 \times D^3 \rightarrow S^1 \times S^3$ be a smooth embedding (not necessarily standard). Consider the identification space

$$M = (S^1 \times S^3 - i(S^1 \times \text{Int}(D^3))) \cup_h (D^2 \times S^2),$$

where h is the identity map of

$$S^1 \times S^2 = \partial(D^2) \times S^2 = \partial(S^1 \times S^3 - i(S^1 \times \text{Int}(D^3))).$$

Show that $\pi_1(M, *)$ is cyclic.

- (9) Let X and Y be metric spaces and let X be compact. Let f be an isometry of X onto a subspace of Y and let g be an isometry of Y onto a subspace of X . Show that f is onto.
- (10) Compute the homology of the space formed as the union of the unit sphere $\{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ and the closed interval along the z -axis from $(0, 0, -1)$ to $(0, 0, 1)$.