UNIVERSITY OF MICHIGAN

DEPARTMENT OF MATHEMATICS

Qualifying Review Examination in Analysis

6 January 2009: Morning Session, 9:00-12:00

1 Suppose that $f \in L^p(R), 1 \leq p < \infty$. Let $T_r(f)(t) = f(t - r)$. Show that $\lim_{r \to 0} \|T_r f - f\|_{L^p} = 0$.

2 Calculate the integral

$$\int_C \frac{6z^5 + 1}{z^6 + z + 1} dz$$

where C is the circle centered at 0 of radius 2, traversed counterclockwise.

3 Find a conformal map from $D = \{z; 0 < \arg z < 2\pi\}$ to $\Omega = \{w; 0 < \text{Im } w < \pi\}$

4 Suppose $1 \leq p < q \leq \infty$. Show that $L^p([0,1]) \supset L^q([0,1])$ and the inclusion is proper.

5 This consists of 3 quick questions. Don't elaborate too much on the answers. $\Delta = \{|z| < 1\}$.

a) Suppose that f is an analytic function on Δ with $f(1/j) = 1/6, j = 2, 3, \ldots$. Show that f is constant.

b) Show that there is no nonconstant bounded analytic function on $\mathbb{C} \setminus \mathbb{Z}$ but that there is a nonconstant bounded analytic function on $\mathbb{C} \setminus [1, \infty >$

c) Show that there exists no analytic function on $f : \Delta \to \Delta$ with $f(0) = 0$ and $f(1/2) = 3/4$.

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6 January 2009: Afternoon Session, 2:00-5:00

6 Suppose that $f \in L^\infty \cap L^1(\mathbb{R})$.
a) Show that $f \in L^p$ for all $1 < p < \infty$.
b) Show that $\lim_{p\to\infty} \|f\|_{L^p} = \|f\|_\infty$.

7 Evaluate $\int_{-\infty}^{\infty} \frac{dx}{4+x^4}$

8 Suppose that $f(x)$ is a bounded function on $\mathbb{R}$ and suppose that $\lim_{x\to 0} f(x) = A$. Show that $\lim_{n\to\infty} \frac{\sqrt{n}}{\pi} \int_{-\infty}^{\infty} \frac{f(x)dx}{1+nx^2} = A$.

9 Let $A = \{1/2 < |z| < 2\}$. Is there an analytic function f on $\mathbb{C} \setminus \{0\}$ so that the imaginary part $\text{Im}(f) < -1$ on ∂A and $f(1) = 0$. Explain your answer.

10 Suppose that $f \in L^1([0,1])$ and continuous on $(0,1]$. Show that $\lim_{\epsilon\to 0} \int_\epsilon^1 f(x)dx$ converges to $\int_0^1 f(x)dx$ or give an example where the conclusion fails.