

## Department of Mathematics

### Syllabus for Math 592: Algebraic Topology.

1. Fundamental group; Seifert-Van Kampen Theorem; the fundamental group of spheres, projective spaces, lens spaces and surfaces.
2. Theory of covering spaces; homotopy lifting property; existence of universal cover, regular coverings, deck transformations and the correspondence between subgroups of the group of deck transformations and based coverings.
3. Simplicial complexes; subdivision and the simplicial approximation theorem; fundamental group of simplicial complexes and in particular of graphs.
4. Singular and simplicial homology; relative homology and the homology long exact sequence; the induced map of homology; the Mayer-Vietoris sequence; excision; homotopy axiom.
5.  $H_1$  is the abelianized fundamental group.
6. Eilenberg-Steenrod axioms.
7. Brouwer's and Lefschetz's fixed point theorems. As an application of the latter to show that even dimensional spheres do not admit a nowhere vanishing vector field.
8. Euler-Poincaré characteristic.

*Optional topic.*

Poincaré-Hopf theorem for vector fields

Topics listed as optional will not be included on the Qualifying Review Examinations. Other topics may be included in the Qualifying Review Examination even when they are not covered in a particular course.