Department of Mathematics

Syllabus for Math 596: Complex Analysis.

1. *Review of advanced calculus and its application to analytic maps.*
   Metric space, continuity, completeness, compactness, connectedness, topology of uniform convergence on compact sets, derivatives of maps of $\mathbf{R}^n$ to $\mathbf{R}^m$, **R**- and **C**-linear maps, complex differentiability and the Cauchy-Riemann equations, complex derivative and the Jacobian, harmonic functions, harmonic conjugate, differential forms of degree 1 and 2, Stokes's theorem in the plane, Pompeiu's formula, Cauchy's theorem and the Cauchy integral formula.

2. Elementary functions and conformal mappings.
   Elementary functions, conformal mappings, Mobius transformations, the Riemann sphere as an example of a complex manifold, rational functions as holomorphic maps of the Riemann sphere.

3. *Consequences of the Cauchy integral formula*.
   Analytic functions C , space of analytic functions closed under uniform limits on compact sets, Morera and Cauchy-Goursat theorems, power series expansions, zeros of analytic functions, identity principle, maximum principle.

4. *Isolated singularities.*
   Laurent series expansion, classification of isolated singularities, Casorati-Weierstrass theorem, Picard's theorem (statement only), meromorphic functions (also as analytic maps into the Riemann sphere), principal part of a meromorphic function at an isolated singularity, partial fraction expansion.

5. *Residue theorem and applications.*
   Residue of a function at an isolated singularity, residue theorem, applications to evaluation of contour integrals, evaluation of definite integrals, generalized partial fraction expansions, homotopic curves, winding number of a closed curve about a point, the argument principle, Rouche's theorem, fundamental group and simply-connected domains, Jordan curve theorem (statement only).

6. *Riemann mapping theorem.*
   Equicontinuity, Arzela-Ascoli theorem, Schwartz's lemma, Montel's theorem, normal families, Riemann mapping theorem.

Optional topics:
Dirichlet problem for harmonic functions, Poisson integral functions, Jensen's formula, infinite products, Hadamard factorization theorem.

References:
Ahlfors, *Complex Analysis,*

Topics listed as optional will not be included on the Qualifying Review Examinations. Other topics may be included in the Qualifying Review Examination even when they are not covered in a particular course.

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