Department of Mathematics

Syllabus for Math 591: General and Differential Topology.

It is expected that about half of the semester will be devoted to general topology and the rest to differential topology.

General Topology

The first two topics should be reviewed in some detail—the pace will depend on the preparation of the class.

1. Definition of topology, continuity, homeomorphism, subspaces, separation axioms, bases and countability axioms. Discuss metric spaces as examples. (It is likely that most of the students would have seen metric spaces before. It would be helpful to relate the abstract concepts to what they already know for metric spaces.)
2. Compactness and local compactness; one point compactification. Lebesgue number Sequences and their convergence. Equivalence (for metric spaces) of compactness with the condition that every sequence has a convergent subsequence.
3. Product and quotient topology. Projective spaces, lens spaces, Klein bottle, compact tori and CW-complexes to be introduced as examples. Group actions.
4. Statement of the Tychonoff theorem for arbitrary products; proof of this theorem for finite products.
5. Connectedness and local connectedness, path connectedness. Connected components. Reminder that intervals are connected. Example of the topologist's sine curve to be given. (About a week to be devoted to these topics.)
6. Statement of Urysohn's lemma, and its proof for metric spaces. Tietze's extension theorem and Urysohn's metrization theorem to be mentioned.
7. Topological groups: examples of matrix groups and homogeneous spaces.

Differential Topology

1. Topological manifolds; differentiable manifolds embedded in $\mathbf{R}^n$; examples.
2. Notion of smooth maps between two differentiable manifolds
3. Tangent spaces and the differential of a smooth map; vector fields; a brief discussion of the tangent bundle (as a submanifold of $\mathbf{R}^{2n}$)
4. Submanifolds, immersion, submersion and embedding
5. Inverse and implicit function theorems (without proof)
6. Regular values, submanifolds arising as $f^{-1}(p)$, p a regular value.
7. Sard's theorem without proof
8. Whitney's embedding theorem
9. Transversality: definition and examples. Intersection of transversal submanifolds

Optional topics: Uniform convergence; Ascoli's theorem; Baire category theorem and its applications to homogeneous spaces. Path metric spaces. Paracompactness and the shrinking lemma. Partition of unity. General differentiable manifolds and Whitney's embedding theorem. Brouwer's fixed point theorem for smooth maps (given as in Milnor's "Topology from differentiable viewpoint"). Classification of surfaces in terms of handle decomposition.

Topics listed as optional will not be included on the Qualifying Review Examinations. Other topics may be included in the Qualifying Review Examination even when they are not covered in a particular course. JSM 3/94