Syllabus for Math 556 - Fall 1995

Text

Fourier Analysis and its Applications, G. Folland, Brooks/Cole Publishing Co., 1992

1. Fourier Series (chap. 2)

Fourier series of a periodic function, Bessel's inequality
$PC(a,b)$, $PS(a,b)$, Dirichlet kernel, pointwise convergence
uniform convergence, Gibbs phenomenon
differentiation of Fourier series, relation between smoothness of f and decay of c_n
sine and cosine series
separation of variables

$u_t = ku_{xx},\ 0 \le x \le l,\ u(x,0) = f(x),\ u(0,t) = u(l,t) = 0$
$u_t = ku_{xx},\ 0 \le x \le l,\ u(x,0) = f(x),\ u_x(0,t) = u_x(l,t) = 0$
$u_t = ku_{xx},\ x > 0,\ u(0,t) = f(t)$ (heat conduction below Earth's surface)
$u_{tt} = c^2 u_{xx},\ 0 \le x \le l,\ u(x,0) = f(x),\ u_t(x,0) = g(x),\ u(0,t) = u(l,t) = 0$
(derive d'Alembert's formula from Fourier series)

2. Orthogonal Functions (chap. 3)

inner product, Cauchy-Schwarz inequality, orthogonality
completeness, characterization of an orthonormal basis for $L^2(a,b)$
dominated convergence theorem (statement only)
best approximation in $L^2(a,b)$
self-adjoint differential operators, regular Sturm-Liouville problems
spectral theorm (statement only)
examples

$f'' + \lambda f = 0,\ f(-\pi) = f(\pi),\ f'(-\pi) = f'(\pi)$
$f'' + \lambda f = 0,\ f'(0) = \alpha f(0),\ f'(l) = \beta f(l)$

3. Boundary Value Problems (chap. 4)

1-d heat flow

$u_t = ku_{xx},\ u(x,0) = f(x),\ u_x(0,t) = \alpha u(0,t),\ u_x(l,t) = -\alpha u(l,t)$
(Newton's law of cooling)
$u_t = ku_{xx},\ u(x,0) = f(x),\ u(0,t) = 0,\ u(l,t) = A$ (inhomogeneous b.c.)
$u_t = ku_{xx} + R,\ u(x,0) = 0,\ u(0,t) = u(l,t) = 0$ (radiation)

1-d wave equation

$u_{tt} = c^2 u_{xx} + F,\ \ u(x,0) = f(x),\ u_t(x,0) = g(x),\ u(0,t) = u(l,t) = 0$
$u_{tt} = c^2 u_{xx},\ \ u(x,0) = f(x),\ u_t(x,0) = g(x)$
$u_x(0,t) = u_x(l,t) = 0$ (pipe open at 2 ends)
$u(0,t) = u_x(l,t) = 0$ (pipe open at 1 end, note effect on frequencies)

Dirichlet problem for Laplace equation

$\Delta u = 0$ in D, $u = f$ on ∂D (rectangle, annulus, disk, Poisson kernel for unit disk)

multivariable Fourier series

2-d, 3-d heat flow on rectangular domains
electrostatic potential due to charge in a rectangular volume