

#### 4. Fourier Transform (chap. 7)

convolution, approximate identity

Fourier transform on  $L^1$ , Riemann-Lebesgue lemma, inversion formula

Fourier transform on  $L^2$ , Plancherel theorem

applications

heat kernel

Dirichlet problem on half-space, Poisson kernel

signal analysis, band-limited functions, sampling theorem

Heisenberg inequality

Sturm-Liouville problems on an unbounded domain, sine and cosine transform

$u_t = ku_{xx}$ ,  $0 < x < \infty$ ,  $u(x, 0) = f(x)$ ,  $u_x(0, t) = 0$  (interpretation via images)

multivariable Fourier transform

discrete Fourier transform, FFT

#### 5. Generalized Functions (chap. 9)

test functions,  $L^1_{loc}$

distributions, delta function

weak convergence, approximation by convolution

weak solution of differential equations

$$u' = 0, \quad xu' = 0, \quad x^2u' = 0$$

$$u_t + cu_x = 0$$

#### 6. Green's Functions (chap. 10)

construction of  $G(x, y)$  for  $Lu = a_2(x)u'' + a_1(x)u' + a_0(x)u$ ,  $B_0u = B_1u = 0$

examples

$$Lu = u'', \quad u(0) = u(1) = 0$$

$$Lu = u'', \quad u'(0) = u'(1) = 0 \text{ (counterexample, state Fredholm alternative)}$$

$$Lu = u'' + \alpha^2u, \quad u(0) = u(1) = 0$$

$$Lu = u'' - \alpha^2u, \quad u(\pm\infty) = 0$$

eigenfunction expansions for  $G(x, y)$ ,  $\delta(x)$

fundamental solution of Laplace equation in 2-d, 3-d

Note: there are many more examples contained in the exercises for these chapters.

#### References

*Methods of Mathematical Physics* R. Courant & D. Hilbert, John Wiley, 1963 & 1966

*Fourier Series and Integrals* H. Dym & H.P. McKean, Academic Press, 1972

*Green's Functions and Boundary Value Problems* I. Stakgold, John Wiley, 1979

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