

Department of Mathematics

Syllabus for Math 597: Real Analysis.

1. *Lebesgue measure in \mathbf{R}^n*
Lebesgue measure on the real line, abstract measure theory, outer measures, measurable sets, class of measurable sets, Borel sets, existence of nonmeasurable sets.
2. *Measurable functions.*
Structure of measurable functions, Littlewood's three principles. Lusin-Vitali theorems, convergence of measurable functions, Egoroff's theorem.
3. *Integration.*
Lebesgue theory, monotone convergence theorem, Fatou's lemma, class L^1 of integrable functions, Holder and Minkowski inequalities, L^p spaces, Fubini and Tonelli theorems, integration by parts and weak derivatives of functions of bounded variation, comparison of Lebesgue-Stieltjes and Riemann-Stieltjes integrals.
4. *Differentiation theory.*
Pointwise derivatives of functions of bounded variation, absolutely continuous functions, maximal functions, fundamental theorem of calculus.
5. *Fourier Analysis.*
Convergence of Fourier series of smooth L^2 functions, Fourier transform, differentiation under the integral.
6. *Optional topics.*
Signed measures, Hahn and Jordan decompositions, Radon-Nikodym theorem, linear functionals, dual of L^p spaces, applications to probability theory, Sobolev spaces and the Sobolev embedding theorem.

References:

Royden, *Real Analysis*,

Natanson, *Theory of Functions of a Real Variable*,

Dym and McKean, *Fourier Series*

Topics listed as optional will not be included on the Qualifying Review Examinations. Other topics may be included in the Qualifying Review Examination even when they are not covered in a particular course.