Polynomials, permutations, and the Paris metro

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Suppose I have several polynomials $p_1(x), p_2(x), \ldots, p_n(x)$, which for small negative values of $x < 0$ satisfy $p_1(x) > p_2(x) > \cdots > p_n(x)$, so their graphs are neatly stacked in order. Then at $x = 0$ catastrophe happens: all of the polynomials satisfy $p_i(0) = 0$ and their graphs all collide at the origin in spectacular fashion.

A moment later, for small $x > 0$, the graphs split apart once again but in some new order $p_{a_1}(x) > p_{a_2}(x) > \cdots > p_{a_n}(x)$.

What can we say about this new ordering? What does this have to do with the Paris Metro?