Constructing the Real Numbers

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Real numbers are so prominent in math today that it's easy to take them for granted. How are we so sure that real numbers exist? Are they even formed on some basis of rigorous mathematical logic? Or maybe we just made them up, and everything we've done so far just magically worked out...

There is in fact a way to rigorously construct the real numbers! In this talk, we'll discuss one construction to convince ourselves that the real numbers are actually... well, real! But in order to do that, we'll need to rigorously construct the integers and the rational numbers first, and that requires more strange symbols and careful details than you may think. We'll even reveal the bare minimum that must be assumed about natural numbers in order to rigorously form the rest of our mathematical universe!