In 1917 Kakeya asked the following question: what is the area of the smallest possible set in which a unit line segment can be turned to face in any direction? Incredibly, the answer to this question turns out to be that such a set can be made to have area $\varepsilon$, for any positive number $\varepsilon$. The proof is surprisingly elementary, and underlies several important theorems in harmonic analysis.