### Thursday, August 04, 2022

1:00pm-3:00pm  **Dissertation Defense** -- Jason Liang (UM) *Vinberg Representations and 2-Descent on Jacobians of Curves* -- https://umich.zoom.us/j/94685587861  Passcode: 234244 Virtual

### Friday, August 05, 2022

12:00pm-2:00pm  **Dissertation Defense** -- Robert Cochrane (UM) *Tensor Ranks and Norms* -- Zoom: https://umich.zoom.us/j/93358265123?pwd=eWF0NFpnblF1VGpNYUZ2U1FqUTHzZz09 Password: tensors Virtual
Dissertation Defense
Thursday, August 04, 2022, 1:00pm-3:00pm
https://umich.zoom.us/j/94685587861 Passcode: 234244 Virtual
Jason Liang (UM)

Vinberg Representations and 2-Descent on Jacobians of Curves

This dissertation applies Vinberg theory to the problem of constructing 2-descent maps on the Jacobians of hyperelliptic curves. In the first part, we construct, given a hyperelliptic curve $C$ with certain marked points $P$ and tangent vectors $t$, a smooth, complete surface $S$. Using the Picard group of $S$, we obtain a 2-graded simple, split adjoint Lie group $(H, \theta)$ of Dynkin type $A_n$ where $n$ depends on the genus of $C$ and the nature of the marked points. We show that there exists an injective 2-descent map from the Jacobian of $C$ into the orbit spaces of the local and global Vinberg representations associated to $(H, \theta)$. The approach is based on previous work of Thorne which addresses the cases where $H$ is of Dynkin type $E_6$ or $E_7$.

In the second part, we construct, given a polynomial $f(x)$ of odd degree $n$ and satisfying certain properties, an explicit map $\psi$ from $J(C)$ into the orbit space of $G = \SO(V) \times \SO(V)$ acting on $\End(V)$, where $C$ is the smooth, complete curve satisfying the equation $y^2 = f(x)$, and $V$ is an orthogonal space of dimension $n$ with maximal index of isotropy and discriminant $1$. The pair $(G,V)$ is exactly the degree 2 Vinberg representation of Dynkin type $D_n$. The map $\psi$ extends a previous construction of Thorne which gives an explicit descent map for the degree 2 Vinberg representation of Dynkin type $A_n$.

In both parts, we work over an arbitrary field $K$ of characteristic 0.

Jason's advisor is Wei Ho.
Tensor decompositions have been studied for nearly a century, but the well-known notion of tensor rank does not capture all the properties that one may desire of a rank function on tensors. For that reason, in recent years many alternative notions of tensor rank have been developed and studied. Some have also studied related notions of norms on tensors, especially the nuclear norm, which can be viewed as a convex relaxation of tensor rank.

In this thesis, we calculate the value of the recently introduced $G$-stable rank for all weights on $2 \times 2 \times 2$ and $2 \times 2 \times 3$ complex-valued tensors, and introduce X-rank, which can be viewed as a refinement of $G$-stable rank.

We then investigate the nuclear norm, and how it and some other norms on tensor product spaces behave with respect to the vertical, or Kronecker, tensor product.