Tuesday, June 01, 2021

11:00am-1:00pm  **Special Events**  -- Devlin Mallory (UM)  *Dissertation Defense: Singularities of Birational Geometry via Arcs and Differential Operators*  --  Virtual

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Wednesday, June 02, 2021

10:00am-12:00pm  **Special Events**  -- Eamon Quinlan (UM)  *Dissertation Defense: Bernstein-Sato theory in positive characteristic.*  --  Virtual
We study singularities of algebraic varieties, and in particular those arising in birational geometry, from several points of view. The first is that of arc schemes: arc schemes parametrize \( \mathbb{A}^n \)-infinitesimal curves on a variety, and their geometry reflects properties of singularities. We show that morphisms of arc schemes (more precisely, of "local" arc schemes) can detect local isomorphisms of varieties; more precisely, we use the triviality of a certain ideal-closure operation to show that if a morphism induces an isomorphism of local arc schemes then it must be an isomorphism on local rings.

We then use arc schemes, in conjunction with the theory of determinantal rings, to verify the semicontinuity conjecture for the behavior of the minimal log discrepancy (a subtle invariant of singularities) in the case of determinantal varieties. In particular, we calculate the Nash ideal of a generic square determinantal variety, which then allows us to give an explicit formula for the minimal log discrepancies of pairs of determinantal varieties and determinantal subvarieties. This allows us to verify the semicontinuity conjecture for such pairs.

We then take another point of view, via the study of differential operators on singular rings. At least since the work of Levasseur and Stafford in the 1980s, the question had been asked of whether one can characterize singularities of rings via certain properties of their rings of differential operators. In particular, one question is whether a ring with mild singularities is a simple module under the action of its ring of differential operators. While an answer in characteristic \( p \) had been provided by Karen Smith, no answer had been forthcoming in characteristic 0. We provide a counterexample showing that the expected connection does not exist, through the study of the global geometry of Fano varieties. More specifically, we show that certain del Pezzo surfaces do not have big tangent bundles, and thus their homogeneous coordinate rings are not simple under the action of their rings of differential operators, despite having "mild" singularities.

Devlin's advisor is Mircea Mustata.

Zoom (passcode is embedded)
https://umich.zoom.us/j/93470523384?pwd=NFe0d01yQ3c1V2hDTW1pV3czZ1hLZz09
Given a holomorphic function $f$ in $n$ variables, its Bernstein-Sato polynomial is a classical invariant that detects the singularities of the zero locus of $f$ in very subtle ways; for example, its roots recover the log-canonical threshold of $f$ and the eigenvalues of the monodromy action on the cohomology of the Milnor fibre. In my thesis I continue the work of Bitoun and Mustață to develop an analogue of this invariant in positive characteristic. More concretely, I develop a notion of Bernstein-Sato polynomial for arbitrary ideals (which, over the complex numbers, was done by Budur, Mustață and Saito), I show that its roots are always rational and negative and that they encode some information about the F-jumping numbers. I also prove that for monomial ideals one can recover the roots of the classical Bernstein-Sato polynomial from this characteristic-p version.

Eamon's advisor is Karen Smith.

Zoom: https://umich.zoom.us/j/94087016623
Password: bfunction